The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b). ✓
2. Find the values of f at the endpoints of the interval. ✓
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value. ✓

48. $f(x) = \underline{5 + 54x - 2x^3}$, $[0, 4]$

Find the absolute maximum & minimum values of $f(x)$ on $[0,4]$.

Step 1. $f'(x) = 54 - 6x^2$

$0 = 54 - 6x^2$ $\quad (\div 6$

$0 = 9 - x^2$

$x = \pm 3$ but only $+3$ is inside $[0,4]$

$f(3) = 5 + 54(3) - 2(3)^3$ remember no calc. on exams.

$= 5 + 54(3) - 2(27)$

$= 5 + 54(3) - 54 = 5 + 54(2) = 5 + 108 = 113$ maximum

Step 2 $f(0) = 5 + 0 = 5$ minimum.

$f(4) = 5 + 54(4) - 2(4)^3$

$= 5 + 4(54 - 2(4)^2) = 5 + 4(54 - 32) = 5 + 4(22)$

$= 93$

Step 3 abs. max is 113 @ $x = 3$

abs. min is 5 @ $x = 0$

Example Find the abs. max & min of $f(x) = \underline{x - \ln x}$ on $[\frac{1}{2}, 2]$

Remark: A calculator is needed for this example. It will not appear on an exam.

Step 1 $f'(x) = 1 -$ $\quad 0 = 1 -$ $\quad$ critical

It will ~~not~~ appear on an exam.

Step1 $f'(x) = 1 - \frac{1}{x}$ $\quad 0 = 1 - \frac{1}{x}$ critical point $x = 1$

$f(1) = 1 - \ln 1 = 1$ min

Step2 Endpoints

$f(\frac{1}{2}) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$ calc.

$f(2) = 2 - \ln 2 \approx 1.31$ max.

Step3 Min is 1 @ 1

max is $2 - \ln 2 \approx 1.31$ @ $x = 2$.

70. After an antibiotic tablet is taken, the concentration of the antibiotic in the bloodstream is modeled by the function

$$C(t) = 8(e^{-0.4t} - e^{-0.6t})$$

where the time t is measured in hours and C is measured in $\mu g/mL$. What is the maximum concentration of the antibiotic during the first 12 hours?

Interval $= [0, 12]$

Step1

$C'(t) = 8(-0.4e^{-0.4t} + 0.6e^{-0.6t})$

$0 = 8(-0.4e^{-0.4t} + 0.6e^{-0.6t})$

$\div 8)$ $\quad 0 = -0.4e^{-0.4t} + 0.6e^{-0.6t}$

$0.4e^{-0.4t} = 0.6e^{-0.6t}$

$\div 0.6)$ $\quad \frac{0.4}{0.6}e^{-0.4t} = e^{-0.6t}$

$\times e^{0.4t})$ $\quad \frac{2}{3} = \frac{0.4}{0.6} = e^{(-0.6+0.4)t} = e^{-0.2t}$

$\frac{2}{3} = e^{-0.2t}$

take ln) $\quad \ln\frac{2}{3} = -0.2t$

$\times -5$ $\quad -5\ln\frac{2}{3} = t$

$C(-5\ln\frac{2}{3}) = 8(e^{-0.4(-5\ln\frac{2}{3})} - e^{-0.6(-5\ln\frac{2}{3})}) \approx 1.19$ Calc.

Step2

The biggest.

Step 2

endpoints $\begin{cases} C(0) = 0 \\ C(12) \approx 0.06 \end{cases}$

Step 3 Maximum concentration is 1.19 $\mu g/mL$