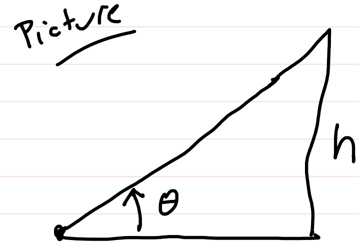
Related rates II

Saturday, March 21, 2015 1:35 PM

A helium balloon is released and rises at a rate of 3m/s. The balloon is tracked by an observer 100m north of point of release of the balloon. How fast is the angle of elevation of the balloon increasing when it is 100m above the ground?

Picture



Known

h = height above ground $\qquad \frac{dh}{dt} = 3\,m/s$

Unknown

θ = angle of elevation

$\frac{d\theta}{dt} = ?$

Equations

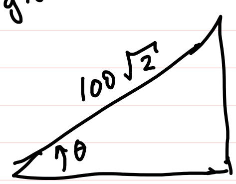
$\tan\theta = \frac{h}{100} = \left(\frac{1}{100}\right)\cdot h$

Solution: $\frac{d}{dt}$ (via chain rule)

$(\sec^2\theta)\frac{d\theta}{dt} = \left(\frac{1}{100}\right)\frac{dh}{dt}$

$(\sec^2\theta)\frac{d\theta}{dt} = \frac{3}{100}$

When the balloon is 100m above ground:



$\sqrt{100^2+100^2} = \sqrt{2\cdot 100^2}$

$= 100\sqrt{2}$

$\cos\theta = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sec\theta = \sqrt{2}$

$\sec^2\theta = 2$

$2\frac{d\theta}{dt} = \frac{3}{100}$

$\frac{d\theta}{dt} = \frac{3}{200}$ rad/s.

22. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

Known

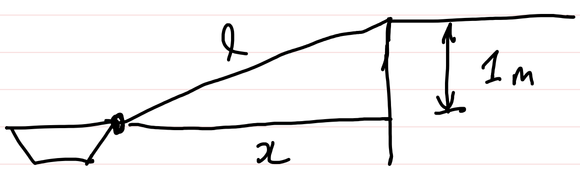
l = length of rope

$\frac{dl}{dt} = -1\,m/s$

Unknown $\quad x$ = distance from dock

$\frac{dx}{dt} = ?$

Picture



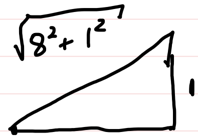
Equation

$l^2 = 1 + x^2$

Solve $\quad 2l\cdot\frac{dl}{dt} = 0 + 2x\frac{dx}{dt}$

When $x = 8$

$\sqrt{65} = \sqrt{8^2+1^2}$



$l = \sqrt{65}$

$x = 8$

$\frac{l}{x}\frac{dl}{dt} = \frac{dx}{dt}$

$-\frac{l}{x} = \frac{dx}{dt}$

x $\overline{dt}$

$-\frac{\sqrt{65}}{8}$ m/s = Answer

More precisely it is approaching the dock @ $\frac{\sqrt{65}}{8}$ m/s.