Example Suppose $4x^2 + 9y^2 = 36$ and $\frac{dy}{dt} = \frac{1}{3}$

Find $\frac{dx}{dt}$ when $x = 2$ and $y = \frac{2}{3}\sqrt{5}$

differentiat with respect to t

i.e apply $\frac{d}{dt}$

How to diff^n $z = 4x^2$ w.r.t t?

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt}$$

$$= 8x \cdot \frac{dx}{dt} \checkmark$$

What about $w = 9y^2$?

$$\frac{dw}{dt} = \frac{dw}{dy} \cdot \frac{dy}{dt}$$

$$= 18y \cdot \frac{dy}{dt}$$

Lets put this together now. $\frac{d}{dt}$ of $4x^2 + 9y^2 = 36$ gives:

$$8x\frac{dx}{dt} + 18y\frac{dy}{dt} = 0$$

sub in circles:

$$8(2) \cdot \frac{dx}{dt} + 18\left(\frac{2}{3}\sqrt{5}\right) \cdot \frac{1}{2} = 0$$

$$16\frac{dx}{dt} + 4\sqrt{5} = 0$$

$$\frac{dx}{dt} = -\frac{4\sqrt{5}}{16} = -\frac{\sqrt{5}}{4}$$

Example The radius of a circle increase at a rate of 2cm/s. How fast is the area of the circle increasing when the radius is 30 cm?

r = radius of circle. It is a function of time!

$r = 30$ only for an instant in time

relate 2 variables.

A = area of circle

$A = \pi r^2$ ← differentiate wrt to t = time

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$ (Chain rule!)

Now we plug in $r = 30$ cm

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ (Chain rule!) $r = 30$ cm

Answer $= 2\pi(30)\cdot(2)$ cm^2/s $\qquad \frac{dr}{dt} = 2$ cm/s

$= 120\pi$ cm^2/s

↖ why are these the units?

no units $\nearrow$ $2\pi\ (30)\ (2)$ $\leftarrow$ cm/s gives cm^2/s

$\leftarrow$ cm

Method for solving related rates problems
(a) What quantities are given in the problem? (give them names)
(b) What is the unknown? (give them names)
(c) Draw a picture of the situation for any time t.
(d) Write an equation that relates the quantities.
(e) Finish the problem (use the chain rule and plug in).

Example A boy is flying a kite 10m above the ground. The kite is moving away from the boy at 1m/s in a horizontal direction. (1m/s parallel to the ground). How fast is the kite moving away from the boy, i.e how fast is the length of string from the boy to the kite increasing when it is 12 m away from the boy?

Givens x = horizontal distance of kite from boy.

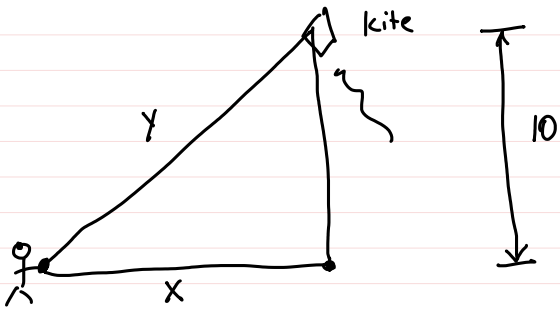
$\frac{dx}{dt} = 1$ m/s

height = 10 m (never changes)

Unknown y = distance from boy to kite

$\frac{dy}{dt}$ = what we have to find.

Picture



Relation $y^2 = 10^2 + x^2$

Solution ↑ diff. w.r.t to time

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

12 ↑ unknown ↑ need to compute this when $y = 12$ ↑ 1m/s

Sub into the relation $y = 12$

$y^2 = 10^2 + x^2$

$12^2 = 10^2 + x^2$

$y^2 = 10^2 + x^2$

$12^2 = 10^2 + x^2$

$144 = 100 + x^2$

$\sqrt{44} = x$ (x = distance so $x \neq -\sqrt{44}$)

now to compute this when $y = 12$, $x = \sqrt{44}$

Sub in given:

$$24 \cdot \frac{dy}{dt} = 2\sqrt{44} \quad (1)$$

$$\frac{dy}{dt} = \frac{2\sqrt{44}}{24}$$

$$= \frac{\sqrt{44}}{12}$$

$$= \frac{\sqrt{4 \times 11}}{12} = \frac{2\sqrt{11}}{12} = \frac{\sqrt{11}}{6}$$

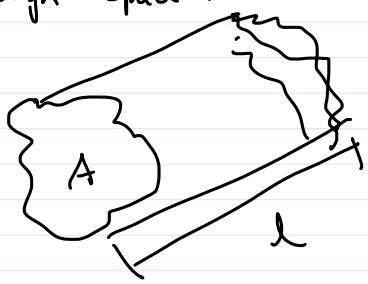
Some useful formulas for Homework (they will be given to you if needed on the exam)

V = volume of sphere $= \frac{4}{3}\pi r^3$

S = surface area of a sphere $= 4\pi r^2$

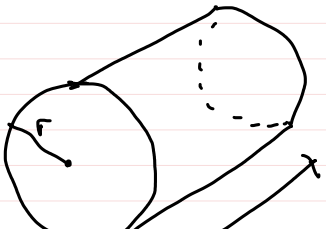
Some formula you should know, i.e not give to on an exam:

Volume of a solid formed by translating a region through space:

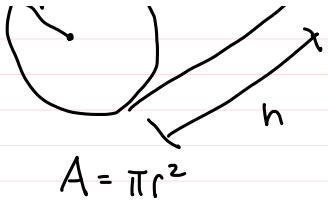


$A \cdot l$ = volume

We apply this to find the volume of a cylinder:



$V = \pi r^2 \cdot h$.



$V = \pi r^2 \cdot h$.

6. The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm?

Known : r = radius

$\frac{dr}{dt} = 4\ mm/s$

diameter = 80 mm

$r = \frac{40}{} = 80/2$ ← Plug in later

unknown $\hat{r}$ V = volume

$\frac{dV}{dt} = ?$

~~picture~~ Equation $V = \frac{4}{3}\pi r^3$

Solution $\frac{dV}{dt} = \frac{4}{3}\pi (3r^2)\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi (40)^2 \cdot 4\ mm^3/s$

$= 16\pi (40)^2\ mm^3/s$

no calc.