Derivatives of logarithms

Tuesday, March 17, 2015 9:04 PM

$y = \ln x \longrightarrow \boxed{e^y = x}$ ← use this

$y' = \frac{1}{x}$

Implicitly diff^n derivative of this eqn

$e^y y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{x} \quad (e^y = x)$

$z = e^y$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$z' = e^y \cdot y'$

$$\boxed{\frac{d(\ln x)}{dx} = \frac{1}{x}}$$

← know me!!

Eg $y = x \ln x \qquad y' = ?$

$u = x \qquad v = \ln x$

$u' = 1 \qquad v' = \frac{1}{x}$ ← Just computed

$y' = x\left(\frac{1}{x}\right) + \ln x$

$= 1 + \ln x$

Remember log rules:

$\ln(AB) = \ln A + \ln B$

$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

$\ln A^c = c \ln A$

Eg $y = \ln(x \sin^2 x) = \ln x + \ln \sin^2 x = \ln x + 2 \ln \sin x$

$y' = ?$

$y' = \frac{1}{x} + 2 \cot x$

$z = \ln \sin x = \ln u \qquad u = \sin x$

$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx}$

$$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Important example for later

$y = \ln|x|$ $\qquad$ $y' = ?$

$$y = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$

$$y' = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ \frac{1}{x} & \text{if } x < 0 \end{cases} \quad \text{same}$$

$z = \ln(-x) = \ln u$ $\qquad$ $u = -x$

$$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot (-1) = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$$

Summary:

$$\boxed{\frac{d \ln|x|}{dx} = \frac{1}{x}}$$

Important later.

E.g

$$y = \ln\sqrt{\frac{1-x^2}{1+x^2}} = \ln\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}} = \frac{1}{2}\left[\ln(1-x^2) - \ln(1+x^2)\right]$$

$y' = ?$

$$y' = \frac{1}{2}\left(\frac{-2x}{1-x^2} - \frac{2x}{1+x^2}\right)$$

similar calculation

$$z = \ln(1-x^2) = \ln u \qquad u = 1-x^2$$

$$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot (-2x) = \frac{-2x}{1-x^2}$$

E.g

$$y = \ln(f(x)) \qquad u = f(x)$$

$$y = \ln u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot f'(x)$$

$$= \frac{f'(x)}{f(x)}$$

Summary

$$\boxed{\frac{d}{dx}\left(\ln(f(x))\right) = \frac{f'(x)}{f(x)}}$$

useful.

Eg

$$y = \frac{\ln x}{1+\ln x}$$

prepare yourself for a long calclulation!

what is y''?

quotient rule $\dfrac{d\left(\frac{u}{v}\right)}{dx} = \dfrac{u'v - v'u}{v^2}$

quotiet rule

$u = \ln x \qquad v = 1 + \ln x$

$u' = \frac{1}{x} \qquad v' = \frac{1}{x}$

$$y' = \frac{\frac{1}{x}(1+\ln x) - \frac{1}{x}\ln x}{(1+\ln x)^2} = \frac{\frac{1}{x}(1+\cancel{\ln x} - \cancel{\ln x})}{(1+\ln x)^2}$$

use product to find

$$y' = \frac{1}{x(1+\ln x)^2}$$

$$y' = \frac{1}{x(1+\ln x)^2}$$

could use product rule now to find y''

$$\longrightarrow \quad y' = \left(\frac{1}{x}\right)(1+\ln x)^{-2}$$

$$u = \frac{1}{x} = x^{-1} \qquad v = (1+\ln x)^{-2}$$

$$u' = \frac{-1}{x^2} \qquad v' = (-2)(1+\ln x)^{-3}\left(\frac{1}{x}\right)$$

chain rule

$$y'' = \frac{-2}{x^2}(1+\ln x)^{-3} - \frac{(1+\ln x)^{-2}}{x^2}$$

$$= \frac{-1}{x^2}\left(\frac{2}{(1+\ln x)^3} + \frac{1}{(1+\ln x)^2}\right)$$

$$= \left(\frac{-1}{x^2}\right)\left(\frac{1}{1+\ln x}\right)^2\left(\frac{2}{1+\ln x} + 1\right)$$

$$= \left(\frac{-1}{x^2}\right)\left(\frac{1}{1+\ln x}\right)^2\left(\frac{2 + 1+\ln x}{1+\ln x}\right)$$