

y a variable, say y depends on u & u depends on x

(e.g. $y = u^2$, $u = \sin x$)

Ultimately y depends on x (e.g. in example $y = \sin^2 x$)

What is $\frac{dy}{dx}$ in terms of $\frac{dy}{du}$ & $\frac{du}{dx}$

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

To remember, imagine the du 's cancelling

Example $y = e^{\tan x}$, use chain rule, introduce new variable u , set $u = \tan x$

$u = \tan x$
 $\frac{du}{dx} = \sec^2 x$
easy.

$y = e^u$
 $\frac{dy}{du} = e^u$

chain rule
 $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\frac{dy}{dx} = e^u \cdot \sec^2 x$$

The answer

$$\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x$$

Example $y = f(x) = \sin(e^x)$

$$f'(x) = ?$$

$y = \sin(e^x)$

Let $u = e^x$

$$y = \sin(e^x)$$

$$y = \sin u$$

$$\frac{dy}{du} = \cos u \quad \checkmark$$

Let $u = e^x$

$$\frac{du}{dx} = e^x \quad \checkmark$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (\cos u) e^x$$

$$= (\cos e^x) e^x \quad \checkmark$$

Example

$$y = \left(t + \frac{1}{t}\right)^5$$

$$\frac{dy}{dt} = y' = ?$$

$$y = \left(t + \frac{1}{t}\right)^5$$

$$y = u^5$$

$$\frac{dy}{du} = 5u^4$$

$$u = t + \frac{1}{t} = t + t^{-1}$$

$$\frac{du}{dt} = 1 - t^{-2}$$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$\frac{dy}{dt} = (5u^4) (1 - t^{-2})$$

$$= 5\left(t + \frac{1}{t}\right)^4 (1 - t^{-2}) \quad \checkmark$$

Answer: $\frac{dy}{dt} = 5\left(t + \frac{1}{t}\right)^4 (1 - t^{-2})$

Example

$$y = \sqrt{u \cos u + e^u}$$

$$\frac{dy}{du} = ?$$

u already used.

$$y = \sqrt{v} = v^{1/2}$$

$$v = u \cos u + e^u$$

product rule!

$$y = \sqrt{v} \quad ' = v^{-1/2}$$

$$V = u \cos u + e^{-u}$$

product rule!

$$\frac{dy}{dv} = \frac{1}{2} v^{-1/2}$$

$$\frac{dV}{du} = \cos u - u \sin u + e^{-u}$$

$$\frac{dy}{du} = \frac{dy}{dv} \cdot \frac{dv}{du}$$

$$V = u \cos u + e^{-u}$$

$$= \frac{1}{2} v^{-1/2} (\cos u - u \sin u + e^{-u})$$

$$= \frac{1}{2} (u \cos u + e^{-u})^{-1/2} (\cos u - u \sin u + e^{-u})$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{u \cos u + e^{-u}}} \cdot (\cos u - u \sin u + e^{-u})$$

$$= \frac{\cos u - u \sin u + e^{-u}}{2 \sqrt{u \cos u + e^{-u}}}$$

Example

$$y = 2^{t^3}$$

$$\frac{dy}{dt} = ?$$

Recall e^t & $\ln t$ are inverse to each other

so: $2 = e^{\ln 2}$ Reverse

$$y = (e^{\ln 2})^{t^3}$$

$$y = e^{(\ln 2)t^3}$$

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

Now: $\ln 2 = \text{some number}!!!$

$$u = (\ln 2)t^3$$

$$\frac{du}{dt} = 3(\ln 2)t^2$$

$$\begin{aligned}
 \frac{dy}{dt} &= \frac{dy}{du} \cdot \frac{du}{dt} \\
 &= e^u \cdot 3(\ln 2)t^2 \\
 &= e^{(\ln 2)t^3} \cdot 3(\ln 2)t^2 \\
 &= (e^{\ln 2})^{t^3} (3 \ln 2) t^2 \\
 &= 2^{t^3} (3 \ln 2) t^2
 \end{aligned}$$

Example

$$y = x^2 e^{-\frac{1}{x}}$$

product rule

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$v = e^{-\frac{1}{x}}$$

$$\frac{dv}{dx} =$$

$$\frac{1}{x^2} e^{-\frac{1}{x}}$$

use chain rule. $(x^{-2} = \frac{1}{x^2})$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{x^2} e^{-\frac{1}{x}} + 2x e^{-\frac{1}{x}} = e^{-\frac{1}{x}} + 2x e^{-\frac{1}{x}}$$

$$v = e^{-\frac{1}{x}} = e^t$$

$$\frac{dv}{dt} = e^t$$

$$t = -\frac{1}{x} = -x^{-1}$$

$$\frac{dt}{dx} = x^{-2}$$

put this up there

$$\frac{dv}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$$

$$= e^t \cdot x^{-2}$$

$$= e^{-\frac{1}{x}} \cdot x^{-2}$$