Example Suppose y is a function of x and satisfies the equation $(y-1)^2 = x$. Find y'.

Sol^n #1 Solve for y

$$y = \pm\sqrt{x} + 1$$

$$y' = \frac{\pm 1}{2\sqrt{x}}$$

These 2 answers are the same. why? $(y-1)^2 = x$ so

$$(y-1) = \pm\sqrt{x}$$

sol^n 2

$$y' = \frac{1}{2(\pm\sqrt{x})} = \frac{\pm 1}{2\sqrt{x}}$$

sol^n 1

Sol^n #2 Start with $(y-1)^2 = x$, & differentiate both sides.

$z = (y-1)^2$ ← LHS.

$$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx} = 2(y-1)\cdot\frac{dy}{dx}$$

differentiating the equation $(y-1)^2 = x$ gives:

$$2(y-1)\frac{dy}{dx} = 1$$

Implicit differentiation

$$y' = \frac{dy}{dx} = \frac{1}{2(y-1)}$$

Example $2x^2 + xy - y^2 = 2$ Harder to solve for y

Find y'. we use implicit differentiation.

Lets look at terms on LHS of the equation and their derivatives

$$2x^2 \xrightarrow{d/dx} 4x$$

$$xy \rightsquigarrow xy' + y$$

product rule

$u = x$ $v = y$

$u' = 1$ $v' = y'$

$$-2y^2 \rightsquigarrow -4yy'$$

$z = -2y^2$

$$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx}$$

$$= -4y \quad y'$$

$$2x^2 + xy - 2y^2 = 2$$

differentiate both sides

$$4x + xy' + y - 4yy' = 0$$

Solve for y'.

Take non y' to the RHS

$$xy' - 4yy' = -4x - y$$

factor out y'

$$y'(x - 4y) = -4x - y$$

$$y' = \frac{-4x - y}{x - 4y}$$

$$y' = \frac{-(4x+y)}{-(4y-x)} = \frac{4x+y}{4y-x}.$$

Example: $xe^y = x - y$ calculate y'.

Diffⁿ both sides wrt ~~y~~ x.

xe^y is a product, so use product rule

$u = x \qquad v = e^y$

$u' = 1 \qquad v' = \frac{dv}{dx} = \frac{dv}{dy} \cdot \frac{dy}{dx} = e^y \cdot \frac{dy}{dx} = e^y \cdot y'$

$\therefore$ Derivative of LHS is $xe^y y' + e^y$

Differentiating. $xe^y = x - y$ gives

$$xe^y y' + e^y = 1 - y'$$

Solve for y', bring y' terms to LHS, non y' terms to RHS

$$xe^y y' + y' = 1 - e^y$$

$$y'(xe^y + 1) = 1 - e^y$$

$$y' = \frac{1-e^y}{1+xe^y}$$

Example $\tan(x-y) = \frac{y}{(1+x^2)}$ calc. y'

1st mult $\times(1+x^2)$ $\quad (1+x^2)\tan(x-y) = y$

LHS $\quad u = (1+x^2) \qquad v = \tan(x-y) = \tan w \qquad w = x-y$

$u' = 2x \qquad v' = \frac{dv}{dx} = \frac{dv}{dw} \cdot \frac{dw}{dx} = \sec^2 w \cdot (1-y')$

$v' = (\sec^2(x-y))(1-y')$

So diffⁿ gives:

$$(1+x^2)\left(\sec^2(x-y)\right)(1-y') + 2x\tan(x-y) = y'$$

Take y''s to RHS

$$(1+x^2)\left(\sec^2(x-y)\right) - y'(1+x^2)\sec^2(x-y) + 2x\tan(x-y) = y'$$

$$(1+x^2)\sec^2(x-y) + 2x\tan(x-y) = y' + y'(1+x^2)\sec^2(x-y)$$

$$(1+x^2)\sec^2(x-y) + 2x\tan(x-y) = y'\left(1 + (1+x^2)\sec^2(x-y)\right)$$

$$\frac{(1+x^2)\sec^2(x-y) + 2x\tan(x-y)}{1 + (1+x^2)\sec^2(x-y)} = y'$$

Example $\quad xe^y = x - y \quad$ calc. y' when $x = 0$

differentiate.

LHS $\quad u = x \quad v = e^y$

$u' = 1 \quad \dfrac{dv}{dx} = \dfrac{dv}{dy}\cdot\dfrac{dy}{dx}$

$\dfrac{dv}{dx} = v' = e^y \cdot y'$

derivative of LHS is

$xe^y y' + e^y$

Now lets diff^c this eq^n

$$xe^y y' + e^y = 1 - y' \qquad \left(y' = \frac{dy}{dx}\right)$$

$$—(\#)$$

Plug in $x=0$ into original.

$$0\cdot e^y = 0 - y$$

$$0 = y$$

point $(0,0)$ plug in here

Set $x=0$ $y=0$ in $(\#)$

$$0\cdot e^0 y' + e^0 = 1 - y' \qquad (\text{Solve for } y')$$

$$1 = 1 - y'$$

$$y' = 0$$