y a variable, say y depends on u & u depends on x

(e.g $y = u^2$, $u = \sin x$)

Ultimately y depends on x (in example $y = \sin^2 x$)

What is $\frac{dy}{dx}$ in terms of $\frac{dy}{du}$ & $\frac{du}{dx}$

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

To remember, imagine the du's cancelling

Example $y = e^{\tan x}$, use chain rule, introduce new variable u, set $u = \tan x$

$u = \tan x$

$\boxed{\frac{du}{dx} = \sec^2 x}$ easy.

$y = e^u$

$\boxed{\frac{dy}{du} = e^u}$

$$\frac{dy}{dx} \overset{\text{chain rule}}{=} \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \cdot \sec^2 x$$

The answer $\boxed{\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x}$

Example $y = f(x) = \sin(e^x)$ $f'(x) = ?$

$y = \sin(e^x)$ Let $\boxed{u = e^x}$

$y = \sin(e^x)$ Let $\boxed{u = e^x}$

$y = \sin u$ $\frac{du}{dx} = e^x$ ✓

$\frac{dy}{du} = \cos u$ ✓

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (\cos u)\, e^x$$

$$= (\cos e^x)\, e^x$$ ✓

<u>Example</u> $y = (t + \frac{1}{t})^5$ $\frac{dy}{dt} = y' = ?$

$y = (t + \frac{1}{t})^5$ $u = t + \frac{1}{t} = t + t^{-1}$

$y = u^5$ $\frac{du}{dt} = 1 - t^{-2}$

$\frac{dy}{du} = 5u^4$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$\frac{dy}{dt} = (5u^4)(1 - t^{-2})$$

$$= 5(t + \tfrac{1}{t})^4 (1 - t^{-2})$$

Answer: $\frac{dy}{dt} = 5(t + \frac{1}{t})^4 (1 - t^{-2})$

<u>Example</u>

$y = \sqrt{(u \cos u + e^u)}$ $\frac{dy}{du} = ?$ u already used!

$y = \sqrt{v} = v^{1/2}$ $v = u \cos u + e^u$

Product rule!

$y = \sqrt{v} = v^{1/2}$

$V = u\cos u + e^{u}$ product rule!

$\frac{dy}{dv} = \frac{1}{2} v^{-1/2}$

$\frac{dV}{du} = \cos u - u\sin u + e^{u}$

$\frac{dy}{du} = \frac{dy}{dv} \cdot \frac{dv}{du}$

$V = u\cos u + e^{u}$

$= \frac{1}{2} v^{-\frac{1}{2}} (\cos u - u\sin u + e^{u})$

$= \frac{1}{2}(u\cos u + e^{u})^{-\frac{1}{2}} (\cos u - u\sin u + e^{u})$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{u\cos u + e^{u}}} \cdot (\cos u - u\sin u + e^{u})$

$= \frac{\cos u - u\sin u + e^{u}}{2\sqrt{u\cos u + e^{u}}}$

Example

$y = 2^{t^3}$ $\qquad \frac{dy}{dt} = ?$

Recall e^{t} & $\ln t$ are inverse to each other

so: $2 = e^{\ln 2}$ Reverse

$y = (e^{\ln 2})^{t^3}$

Now: $\ln 2$ = some number!!!

$y = e^{(\ln 2) t^3}$

$u = (\ln 2) t^3$

$y = e^{u}$

$\frac{du}{dt} = 3(\ln 2) t^2$

$\frac{dy}{du} = e^{u}$

du

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$= e^u \cdot 3(\ln 2)t^2$$

$$= e^{(\ln 2)t^3} \, 3(\ln 2)t^2$$

$$= \left(e^{\ln 2}\right)^{t^3} (3\ln 2)\, t^2$$

$$= 2^{t^3} (3\ln 2)\, t^2$$

Example $y = x^2 e^{-\frac{1}{x}}$ product rule

$u = x^2$ $\qquad$ $v = e^{-\frac{1}{x}}$ use chain rule.

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = \frac{1}{x^2} e^{-\frac{1}{x}}$ $\qquad$ $\left(x^{-2} = \frac{1}{x^2}\right)$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{x^2} e^{-\frac{1}{x}} + 2x e^{-\frac{1}{x}} = e^{-\frac{1}{x}} + 2x e^{-1/x}$$

$v = e^{-\frac{1}{x}} = e^t$ $\qquad$ $\boxed{t = -\frac{1}{x} = -x^{-1}}$

$\frac{dv}{dt} = e^t$ $\qquad$ $\frac{dt}{dx} = x^{-2}$

$$\frac{dv}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$$

$$= e^t \cdot x^{-2}$$

put this up the $\frac{dv}{dx} = e^{-\frac{1}{x}} \cdot x^{-2}$