Derivatives of Trigonometric Functions

Saturday, March 7, 2015 7:34 PM

$f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh + \sinh \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \left(\frac{\cosh - 1}{h}\right) + \left(\frac{\sinh}{h}\right)\cos x$$

2 special limits!

$$= \sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

Remember these!

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

← similar check for yourself

How to check $\frac{d}{dx}\tan x = \sec^2 x$, using

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$\tan x = \frac{\sin x}{\cos x}$ ← use quotient rule!

$u = \sin x$ $\quad v = \cos x$

$u' = \cos x$ $\quad v' = -\sin x$

$$\frac{d(\tan x)}{dx} = \frac{u'v - v'u}{v^2} = \frac{(\cos x)(\cos x) - (-\sin x)(\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$\cos^2 x + \sin^2 x = 1$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

cos x

$= \frac{1}{\cos^2 x} = \sec^2 x$

Example $y = \sin\theta\cos\theta$

$u = \sin\theta$ $\quad$ $v = \cos\theta$

$u' = \cos\theta$ $\quad$ $v' = -\sin\theta$

product rule $= uv' + v'u$

$y' = -\sin^2\theta + \cos^2\theta$

$y' = \cos^2\theta - \sin^2\theta = \cos 2\theta$ ← identity

More Trig derivatives

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

} follow from other 3. See H/W

E.g $y = e^x \sec x$

$u = e^x$ $\quad$ $v = \sec x$

$u' = e^x$ $\quad$ $v' = \sec x \tan x$

$y' = e^x \sec x \tan x + e^x \sec x$

Example $y = e^x \cos x$ find the equation of tangent line @ $(0,1)$

$y' = e^x \cos x - e^x \sin x$

$x = 0$

Slope of tangent $= m = e^0 \cos 0 - e^0 \sin 0$

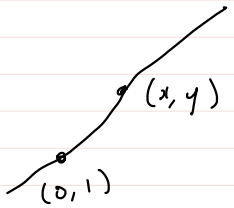
$= 1 - 0 = 1$

Slope $= 1$

point $= (0,1)$

target $= 1 - 0 = 1$

point $= (0,1)$



$$1 = \text{slope} = \frac{y-1}{x-0}$$

$$1 = \frac{y-1}{x}$$

$$x = y - 1$$

$$x + 1 = y$$

E.g. $f(\theta) = \theta \cos\theta$ $\quad f''(\theta) = ?$

product rule

$$f'(\theta) = \cos\theta - \theta\sin\theta$$

$u = \theta \quad v = \sin\theta$

$u' = 1 \quad v' = +\cos\theta$

$$f''(\theta) = -\sin\theta - (\theta\cos\theta + \sin\theta)$$

$$= -2\sin\theta - \theta\cos\theta$$

E.g. For what values of x is the target to $y = e^x \cos x$ horizontal?

$$y' = e^x\cos x - e^x \sin x \quad \text{(product rule)}$$

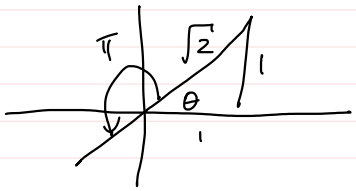
$0 = \text{slope} = e^x\cos x - e^x\sin x$ ← horizontal tangent

$$0 = e^x(\cos x - \sin x) \qquad e^x > 0 \text{ can cancel.}$$

$$0 = \cos x - \sin x$$

$$\sin x = \cos x$$

$$\tan x = \frac{\sin x}{\cos x} = 1$$



$\tan\theta$ π-periodic

$$\tan\theta = 1 \qquad \theta = \frac{\pi}{4}, \frac{\pi}{4}+\pi, \frac{\pi}{4}+2\pi, \ldots \frac{\pi}{4}-\pi, \ldots$$

Answer $x = \ldots \frac{\pi}{4}-2\pi, \frac{\pi}{4}-\pi, \frac{\pi}{4}, \frac{\pi}{4}+\pi, \frac{\pi}{4}+2\pi, + \ldots$