The derivative as a function

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$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{t\to a} \frac{f(t) - f(a)}{t-a}$$

we may vary a and obtain a new function of x, we call it the derivative of f(x).

So

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{t\to x} \frac{f(t) - f(x)}{t-x}.$$

Example $f(x) = x^2$ what is $f'(x)$?

$$f'(x) = \lim_{t\to x} \frac{f(t) - f(x)}{t-x} = \lim_{t\to x} \frac{t^2 - x^2}{t-x}$$

$$= \lim_{t\to x} \frac{\cancel{(t-x)}(t+x)}{\cancel{t-x}}$$

$$= \lim_{t\to x} t+x = 2x$$

Example $f(x) = \sqrt{1-x}$ $f'(x) = ?$

$$f'(x) = \lim_{t\to x} \frac{f(t) - f(x)}{t-x}$$

conjugate!

$$= \lim_{t\to x} \frac{(\sqrt{1-t} - \sqrt{1-x})}{t-x} \cdot \frac{(\sqrt{1-t} + \sqrt{1-x})}{(\sqrt{1-t} + \sqrt{1-x})}$$

$$= \lim_{t\to x} \frac{(1-t) - (1-x)}{(t-x)(\sqrt{1-t} + \sqrt{1-x})}$$

$$= \lim_{t\to x} \frac{(x-t)}{(t-x)(\sqrt{1-t} + \sqrt{1-x})}$$

$$= \lim_{t\to x} \frac{-\cancel{(t-x)}}{\cancel{(t-x)}(\sqrt{1-t} + \sqrt{1-x})} = \lim_{t\to x} \frac{-1}{\sqrt{1-t} + \sqrt{1-x}}$$

$$= \frac{-1}{2\sqrt{1-x}}$$

$$= \frac{}{2\sqrt{1-x}}$$

Example $f(x) = x^3$ $f'(x) = ?$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

expand, could use binomial thm = pascal's Δ.

$$(x+h)^3 = (x+h)(x+h)(x+h)$$

$$= (x+h)(x^2 + hx + hx + h^2)$$

$$= (x+h)(x^2 + 2hx + h^2)$$

$$= x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3$$

$$= x^3 + 3hx^2 + 3h^2x + h^3$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}x^2 + 3h^{\cancel{2}}x + h^{\cancel{3}\,2}}{\cancel{h}} = \lim_{h\to 0} 3x^2 + 3hx + h^2 = 3x^2$$