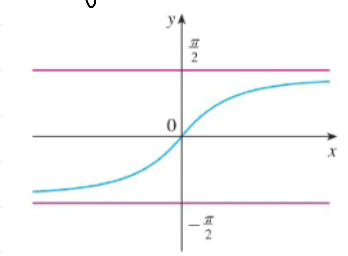
Limits at Infinity : Part II

Tuesday, March 3, 2015 9:02 AM

$y = \tan^{-1} x$



$$\lim_{x \to \infty} \tan^{-1} x = \pi/2$$

$$\lim_{x \to -\infty} \tan^{-1} x = -\pi/2$$

(Change of variables) Substitution for limits: Suppose $\lim_{t \to a} f(t) = b$

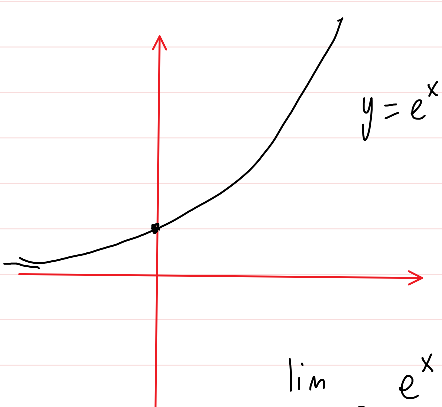
Then $\lim_{t \to a} g(f(t)) = \lim_{x \to b} g(x)$ $\quad x = f(t)$

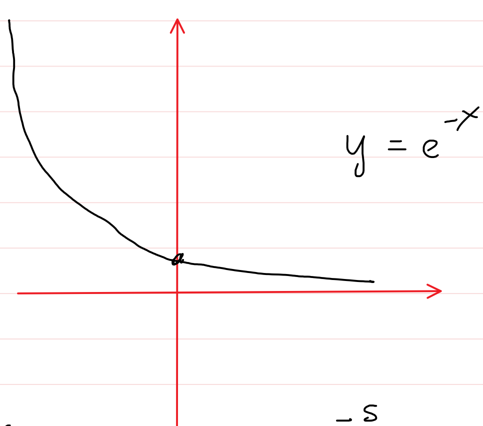
provided all limits exist in the equation.

<u>Example</u>

$$\lim_{t \to 0^+} \tan^{-1}\left(\frac{1}{t}\right) = \lim_{s \to \infty} \tan^{-1}(s) = \pi/2$$

as $t \to 0^+$ $\quad s = \frac{1}{t} \longrightarrow +\infty$





$$\lim_{x \to \infty} e^x = +\infty$$

$x = -s$

$$\lim e^{-s} = 0$$

$$\lim_{x\to\infty} e = +\infty \qquad\qquad \lim_{s\to\infty} e^{-s} = 0$$

$x = -s$

$$\lim_{x\to-\infty} e^{x} = 0 \qquad\qquad \lim_{s\to-\infty} e^{-s} = +\infty$$

<u>Example</u>

$$\lim_{x\to\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x\to\infty} \frac{\frac{e^{3x}}{e^{3x}} - \frac{e^{-3x}}{e^{3x}}}{\frac{e^{3x}}{e^{3x}} + \frac{e^{-3x}}{e^{3x}}}$$

as $x \to \infty$ $(e^{x})^{3}$ is $\curvearrowright$ e^{3x}

the largest term in the denominator so divide through by it

$\cancel{e^{6x}}$

$$= \lim_{x\to\infty} \frac{1 - \cancel{e^{-6x}}^{\to 0}}{1 + \cancel{e^{-6x}}^{\to 0}} = 1.$$

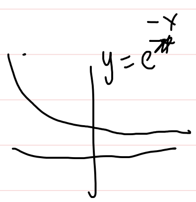
<u>Example</u> $\lim_{x\to\infty} e^{-x}\sin x = 0$ by squeeze thm.

Apply squeeze theorem:

$-1 \le \sin x \le 1$

$\cdot\, e^{-x}$) $\quad -e^{-x} \le e^{-x}\sin x \le e^{-x}$

$$\begin{array}{ccc} -e^{-x} & & e^{-x} \\ \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$



② $$\lim_{x\to\infty} \ln(2+x) - \ln(1+x) = \lim_{x\to\infty} \ln\left(\frac{2+x}{1+x}\right)$$

(2) $\lim_{x \to \infty} \ln(2+x) - \ln(1+x) = \lim_{x \to \infty} \ln\left(\frac{}{1+x}\right)$

$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

$= \lim_{s \to 1} \ln s = \ln 1 = 0$

Change of variables.

continuity of lns..

The inside

$$\lim_{x \to \infty} \frac{2+x}{1+x} = \lim_{x \to \infty} \frac{\frac{2}{x}+1}{\frac{1}{x}+1}$$

$\div x$

$$= \frac{0+1}{0+1} = 1$$