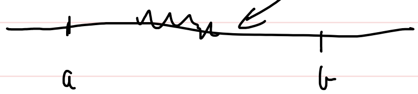
Intermediate Value Theorem

Monday, March 2, 2015 9:22 AM

A function is *continuous* on a closed interval *[a,b]* if it is continuous on *(a,b)*, right continuous at *a* and left continuous at *b.*

ordinary contivity here



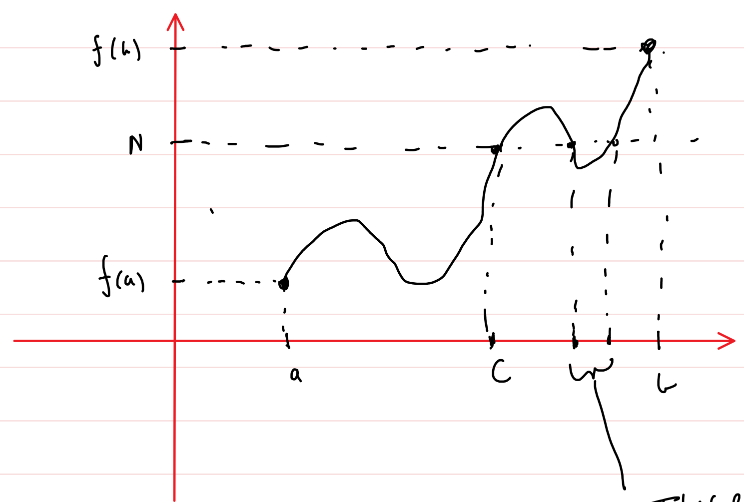
at b: $\lim_{x \to b} f(x)$ may not make sense.

$\lim_{x \to b^-} f(x)$ is OK.

So $x = b$ left continuity is meaningful.

Similarly right continuity at $x = a$.

The Intermediate Value Theorem
Suppose that *f(x)* is continuous on the closed interval *[a,b]* and also assume that $f(a) \neq f(b)$. Let *N* be a number between *f(a)* and *f(b)*. Then there is a *c* in *(a,b)* with $f(c) = N$.



$f(c) = N$

These 2 points would work also.

<u>Example</u> Show that $e^x = -x$ has a root (solution) on $(-1, 2)$

Note: You are $\underline{\underline{\underline{\text{not}}}}$ required to find the solution.

$e^x = -x$

$f(x) = e^x + x$ is continuous on $[-1, 2]$

$f(2) = e^2 + 2 > 0$

$f(-1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0$ as $e^{-1} < 1$ $e = 2.71828...$

By the intermediate value theorem (IVT) there is a c with $0 = f(c) = e^c + c$

N

But this means $-c = e^c$

i.e. c is a root of $e^x = -x$