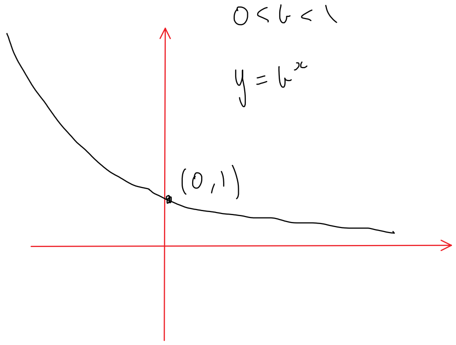
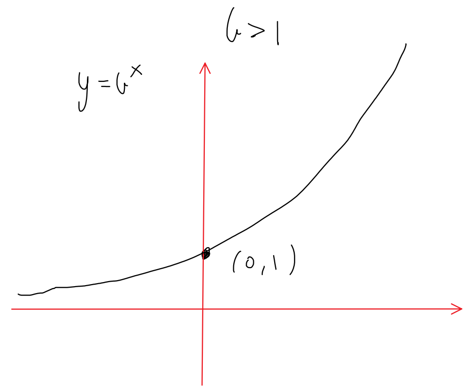
Logarithms

Friday, February 13, 2015 1:29 PM

If $b \neq 1$ then $f(x) = b^x$ $(b>0)$ is 1-to-1





In both cases domain is $(-\infty, +\infty)$ i.e all real numbers
range is $(0, +\infty)$ i.e all positive numbers

The inverse function of $f(x) = b^x$ is the logarithm with base b denoted $\log_b x$.

$$(-\infty, \infty) \underset{\log_b x}{\overset{b^x}{\rightleftarrows}} (0, +\infty)$$

$\log_b x$ has domain $(0, +\infty)$ & range $(-\infty, +\infty)$

Example ① As $b^0 = 1$ always we see that

$\log_b 1 = 0$ ($\log_b x$ is the inverse function of b^x)

② What is $\log_2 8$?

$\log_2 8 = \alpha$ means $2^\alpha = 8$

(as $\log_2 x$ and 2^x are ~~other~~ inverse to each)

As $2^3 = 8$ $\alpha = 3$, So $\log_2 8 = 3$

③ What is $\log_5 \frac{1}{125}$?

~~If~~ $\log_5 \frac{1}{125} = \alpha$ means $5^\alpha = \frac{1}{125}$.

To answer the question we need to find such an α!

$5^3 = 125$ so $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ $\alpha = -3$

Answer: $\log_5 \frac{1}{125} = -3$

Remember no calculators on midterm & Final.

The exponent rules lead to the following log rules

(1) $\log_b(xy) = \log_b x + \log_b y$

(2) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

(3) $\log_b(x^y) = y\log_b x$

Example Write $\ln(5) + 2\ln 2$ as a single logarithm.

Firstly $\ln x = \log_e x$. (natural logarithm)

Lets answer the question:

$\ln(5) + 2\ln 2 = \ln 5 + \ln 2^2$ by 3

$= \ln(5 \times 4)$ by 1

$= \ln 20$.

$f(x) = b^x$ $f^{-1}(x) = \log_b x$

Recall

(1) $x = f(f^{-1}(x))$

(2) $x = f^{-1}(f(x))$

(1) gives $x = f(\log_b x) = b^{\log_b x}$

(2) gives $x = \log_b b^x$

Example (1) Solve for x (2) Solve for x

raise to e^{th} power

$$\ln(x^2-1) = 3$$
$$e^{\ln(x^2-1)} = e^3$$
$$x^2 - 1 = e^3$$
$$x^2 = e^3 + 1$$
$$x = \pm\sqrt{e^3+1}$$

$$e^{5x+1} = 4$$

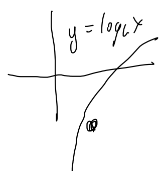
Apply ln)

$$\ln\left(e^{5x+1}\right) = \ln 4$$
$$5x+1 = \ln 4$$
$$5x = \ln 4 - 1$$
$$x = \frac{1}{5}(\ln 4 - 1).$$

Eg Solve the inequality for x

$$1 < e^{3x-1} < 2$$

Interlude If $a>1$ the graph of $y = \log_a x$ is:



dom: $(0, \infty)$
range: $(-\infty, \infty)$

This function is increasing.
Increasing functions preserve inequalities
i.e If $a<b$ then $f(a) < f(b)$

Back to the problem: $1 < e^{3x-1} < 2$
$\ln x$ is increasing
so preserves inequalities

$$\ln 1 < \ln e^{3x-1} < \ln 2$$
$$0 < 3x - 1 < \ln 2$$
$$+1 \qquad 1 < 3x < \ln 2 + 1$$
$$\frac{1}{3} < x < \frac{\ln 2 + 1}{3}$$

Example Solve the inequality for x

$$1 - 2\ln x < 3$$
$$1 < 3 + 2\ln x$$
$$-2 < 2\ln x$$
$$-1 < \ln x$$

(e^x is increasing) e^x)

$$e^{-1} < e^{\ln x}$$
$$e^{-1} < x$$