Limits

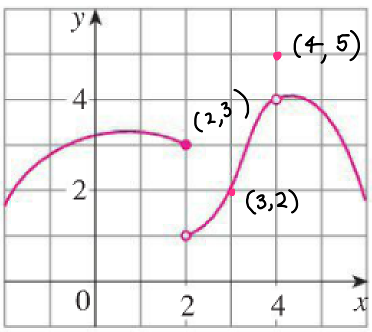
Friday, February 13, 2015 1:30 PM

1 Intuitive Definition of a Limit Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.



[Equation] 2

[Equation] 4

[Equation] does not exist

[Equation] 2

[Equation] 3

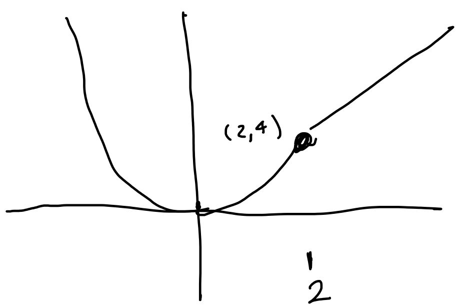
$f(4) = 5$

line

Example $f(x) = \begin{cases} x+2 & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases}$

parabola

$\lim_{x \to 2} f(x) = ?$



$\lim_{x \to 2} f(x) = 4$

2 Definition of One-Sided Limits We write

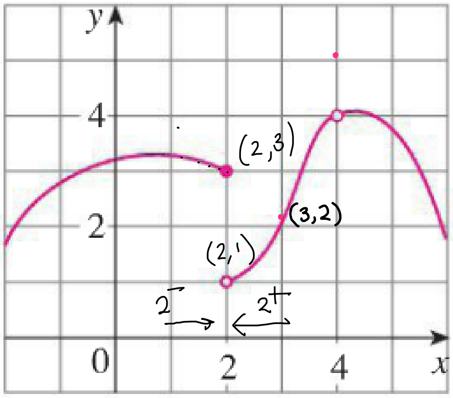
$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

The right hand limit

[Equation]

Is defined similarly



[Equation] 1

[Equation] 3