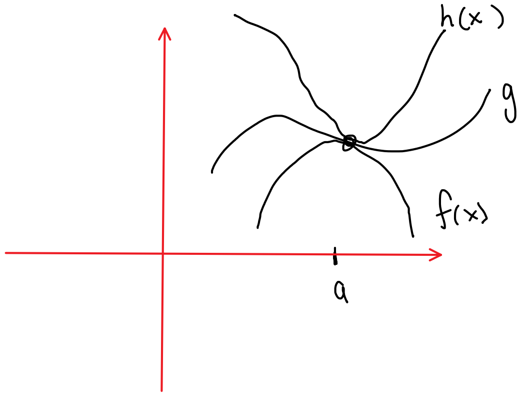
Limit Laws II

Saturday, February 28, 2015 5:19 PM

3 The Squeeze Theorem If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then $$\lim_{x \to a} g(x) = L$$



we want to find $\lim_{x \to a} g(x)$

In the picture:

$$\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$$

then $\lim_{x \to a} g(x) = L$

Example Suppose $\underbrace{5x+1}_{f(x)} \leq g(x) \leq \underbrace{x^2+5x+1}_{h(x)}$ near 0.

What is $\lim_{x \to 0} g(x)$?

$$\lim_{x \to 0} 5x+1 = 1 = \lim_{x \to 0} x^2+5x+1$$

By squeeze: $\lim_{x \to 0} g(x) = 1$.

Sometimes you will have to come up with bounding functions yourself:

Example $\lim_{x \to 0^+} x \sin\left(\frac{1}{x}\right) = 0$

f, h?

Start with $-1 \leq \sin(\text{anything}) \leq 1$

i.e $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$

$f(x) = -x$ $h(x) = x$

$x \to 0^+$

$(x > 0)$

mult. through by x $\downarrow$ $\downarrow$ In squeez thm.

0 0

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Sometimes it is easier to compute 2 1-sided limits for example when $f(x)$ has an absolute value in it.

Reminders on $|x|$.

if $x \geq 0$ then $|x| = x$

What if $x < 0$? $|-7| = 7 = -(-7)$

$|-\pi| = -(-\pi) = \pi$

so if $x < 0$ then $|x| = -x$

In general

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example

$$\lim_{x \to -6} \frac{2x+12}{|x+6|} = \frac{0}{0}$$

Can't plug in.

Break into 2 1-sided limits and apply

$$|x+6| = \begin{cases} x+6 & \text{if } x+6 \geq 0 \\ -(x+6) & \text{if } x+6 < 0 \end{cases}$$

$$\boxed{|x+6| = \begin{cases} x+6 & \text{if } x \geq -6 \\ -(x+6) & \text{if } x < -6 \end{cases}}$$

$$\lim_{x \to -6^+} \frac{2x+12}{|x+6|} = \lim_{x \to -6} \frac{2x+12}{x+6} = 2$$

$x \geq -6$

$$\lim_{x \to -6^-} \frac{2x+12}{|x+6|} = \lim_{x \to -6^-} \frac{2x+12}{-(x+6)} = -2$$

$x < -6$

So $\lim_{x \to -6} \frac{2x+12}{|x+6|}$ does not exist = DNE.

Example

$$\lim_{x \to 1} x + |x-1| = 1$$

$$|x-1| = \begin{cases} x-1 & \text{if } x \geq 1 \\ -(x-1) & \text{if } x < 1 \end{cases}$$

$$\lim_{x\to1^+} x + |x-1| = \lim_{x\to 1} x + x - 1 = 1$$

$$\lim_{x\to1^-} x + |x-1| = \lim_{x\to 1^-} x - (x-1) = 1$$

whole number

<u>Example</u> $[\![x]\!]$ = largest integer smaller that or equal to x

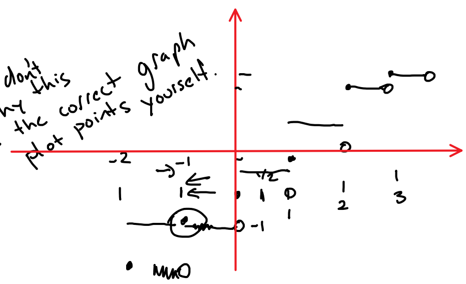
approx.

$[\![2]\!] = 2$ $\qquad \pi \approx 3.14$

$[\![0]\!] = 0$ $\qquad [\![\pi]\!] = 3$

$[\![-1]\!] = -1$ $\qquad [\![\pi]\!] = [\![-3.14\ldots]\!] = -4$

If you don't see why this is the correct graph plot points yourself.



$$\lim_{x\to\frac{1}{2}} [\![x]\!] = 0$$

$$\lim_{x\to-1} [\![x]\!] = DNE$$

$$\lim_{x\to-1^+} [\![x]\!] = -1$$

$$\lim_{x\to-1^-} [\![x]\!] = -2$$