Limit Laws I

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Limit Laws Suppose that c is a constant and the limits

$$\lim_{x\to a} f(x) \quad \text{and} \quad \lim_{x\to a} g(x)$$

exist. Then

1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$
4. $\lim_{x\to a} [f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

Example Suppose $\lim_{x\to 2} f(x) = 3$

$\lim_{x\to 2} g(x) = 4$

What is $\lim_{x\to 2} \frac{\pi f(x) + 3g(x)}{g(x)}$?

Answer $= \frac{\lim_{x\to 2} (\pi f(x) + 3g(x))}{\lim_{x\to 2} g(x)}$ use (5)

$= \frac{\pi \lim_{x\to 2} f(x) + 3 \lim_{x\to 2} g(x)}{4}$ Use (1) & (3)

$= \frac{3\pi + 12}{4}$ ✓

6. $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ where n is a positive integer
7. $\lim_{x\to a} c = c$
8. $\lim_{x\to a} x = a$
9. $\lim_{x\to a} x^n = a^n$ where n is a positive integer
10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer (If n is even, we assume that $a > 0$.)
11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ where n is a positive integer [If n is even, we assume that $\lim_{x\to a} f(x) > 0$.]

Example

$\lim_{x\to 4} x^3 + 5x - 1 = \lim_{x\to 4} x^3 + \lim_{x\to 4} 5x - \lim_{x\to 4} 1$

$= (4)^3 + 5(4) - 1 = 64 - 20 - 1$

$= \ldots$

use (9), (7)

Example $\lim_{x\to 3} \sqrt{x^2+2x+1} = \sqrt{\lim_{x\to 3} (x^2+2x+1)} = \sqrt{9+6+1}$

$= \sqrt{16} = 4$

(11.)

as per last example

Direct Substitution Property If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a)$$

We will not write down all steps in calculations such as the previous 2 examples

We will just plug in

What is a polynomial?

Something like

$5x^4 - 3x^2 + \pi x + 4$

rational function = ratio of 2 polynomials.

Recall

$$5.\ \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}\quad \text{if } \lim_{x\to a}g(x)\neq 0$$

What happens when $\lim_{x\to a} g(x) = 0$!

two possibilities — (1) If $\lim_{x\to a} f(x) \neq 0$ use ideas from the "Infinite limits" lesson to compute

— (2) If $\lim_{x\to a} f(x) = 0$ then we must use some algebraic magic to compute the limit, examples follow!

Example

$$\lim_{x\to -1}\frac{2x^2+3x+1}{x^2-2x-3} = \lim_{x\to -1}\frac{(x+1)(2x+1)}{(x-3)(x+1)} = \lim_{x\to -1}\frac{2x+1}{x-3}$$

(plug in) $= \frac{-1}{-4} = \frac{1}{4}$.

Factor

Factoring the numerator: (a way that you might not know.)

$$\frac{(2x+2)(2x+1)}{2} = \frac{2(x+1)(2x+1)}{2}$$

Advantage of this method: it is quick & fast when the coefficent of x^2 many

$1 \times 2 = 2$ ← Two add numbers multiply to this

Example

$$\lim_{x\to -4}\frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to -4}\left(\frac{\sqrt{x^2+9}-5}{x+4}\right)\cdot\frac{(\sqrt{x^2+9}+5)}{(\sqrt{x^2+9}+5)}$$

Plug in $\frac{0}{0}$

This is just 1

$$= \lim_{x\to -4}\frac{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)}{(x+4)(\sqrt{x^2+9}+5)}$$

Recall multiplying by the conjugate (from "New Identities" lesson).

(leave bottom, expand top as diff. of 2 squares)

$$(a-b)(a+b) = a^2 - b^2$$

↑ difference of 2 squares.

There is no need to show this much work

$$\lim_{x \to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{(x-4)(x+4)}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x-4}{\sqrt{x^2+9}+5}$$

Now just plug in.

$$= \frac{-8}{\sqrt{25}+5} = \frac{-8}{10} = -\frac{4}{5}$$