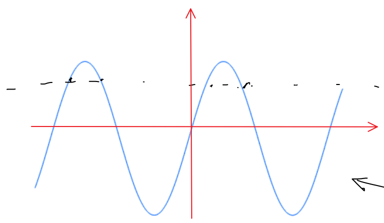
Inverse Trigonometric Functions

Friday, February 13, 2015 1:29 PM

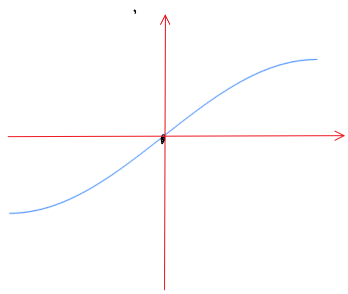
[Equation]



Not 1-to-1
(horizontal line test!)

range $[-1,1]$

[Equation]



Here is graph of sine restricted to $[-\frac{\pi}{2},\frac{\pi}{2}]$

Now it is 1-to-1.

The inverse function is $\sin^{-1}x$ sometimes called arcsine.

$$\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \underset{\sin^{-1}x}{\overset{\text{sine}}{\rightleftarrows}} [-1,1]$$

$\sin^{-1}x$ has domain $[-1,1]$
range $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\sin^{-1}a = b$ means $\sin b = a$
and
$-\frac{\pi}{2} \le b \le \frac{\pi}{2}$

Warning! $\sin^{-1}x \neq \dfrac{1}{\sin x}$

Example What is $\sin^{-1}\left(-\frac{1}{2}\right) = ?$

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

sine : $\text{angles} \underset{}{\overset{\text{sine}}{\rightleftarrows}} \text{numbers}$

arcsine $= \sin^{-1} x$

Think of θ as an angle. $\sin\theta = -\frac{1}{2}$ and

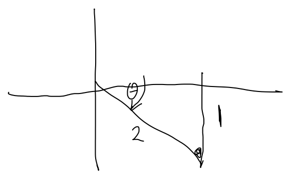
$$\underbrace{-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}_{\theta \text{ is in Quad I or } \underline{IV}}$$

Must be quad $\underline{IV}$ as sine is positive in Quad I.

~~Summary~~ $\sin^{-1}(-\frac{1}{2}) = \theta \rightsquigarrow$ - θ in quad $\underline{IV}$

$\sin\theta = -\frac{1}{2}$ Lets draw it.

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sin\theta = \frac{y}{r} = \frac{-1}{2}$ $\quad y = -1$

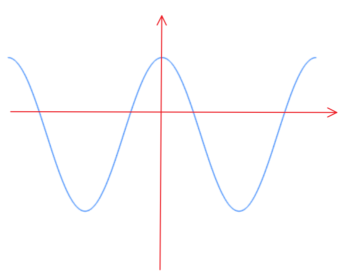
$r = 2$

$\theta = -\pi/6$ by special $\triangle$.

$(\sqrt{3}, -1)$ ← spec. $\triangle$ or pythag.

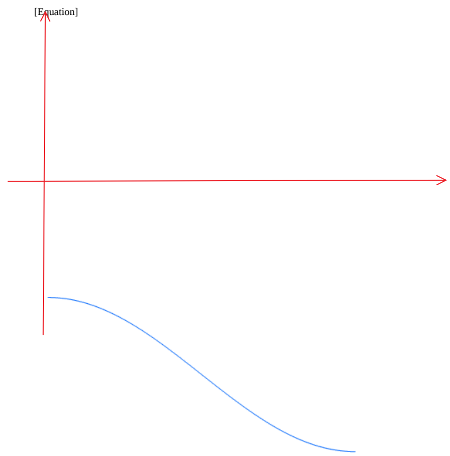
Answer : $-\pi/6$

[Equation]



Not 1-to-1 as before!

[Equation]



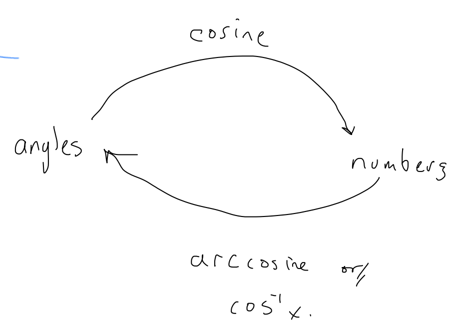
Graph of cosine restricted to $[0, \pi]$.

Is 1-to-1

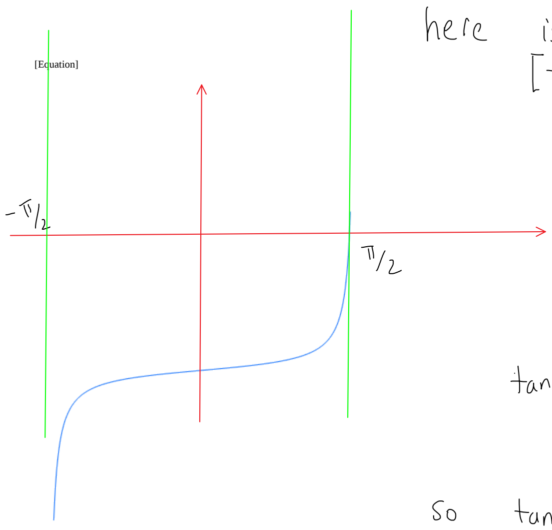
The inverse function $\cos^{-1} x$ or arccosine

$\cos^{-1} x$ has domain $[-1, 1]$

range $[0, \pi]$



Once again $y = \tan x$ is not 1-to-1



here is $y = \tan x$ restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$. This restricted function is 1-to-1

The inverse function is

$\tan^{-1} x$ (or $\arctan x$)

$\tan x$ has ~~do~~ restricted domain $(-\pi/2, \pi/2)$

range $(-\infty, \infty)$

So $\tan^{-1} x$ has domain $(-\infty, \infty)$

range $(-\pi/2, \pi/2)$

Summary

$\cos^{-1} a = b$ means $\cos b = a$ and

$0 \leq b \leq \pi$

$\tan^{-1} a = b$ means $\tan b = a$ and

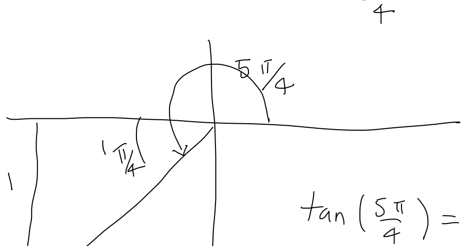
$-\pi/2 < b < \pi/2$

Example Compute $\tan^{-1}\left(\tan \frac{5\pi}{4}\right) =$

Note $\tan^{-1}$(something) is betwen $-\frac{\pi}{2}$ & $\frac{\pi}{2}$

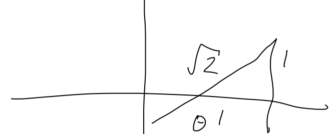
so $\tan^{-1}\left(\tan \frac{5\pi}{4}\right) \neq \frac{5\pi}{4}$.

lets draw $\frac{5\pi}{4}$



$\tan\left(\frac{5\pi}{4}\right) = \frac{y}{x} = 1$

So $\tan^{-1}\left(\tan \frac{5\pi}{4}\right) \overset{\text{used}}{=} \tan^{-1}(1)$



$(-1, -1)$

$\tan^{-1}(1) = \theta$ $\quad \tan\theta = 1$

$= \pi/4$

Example

~~sin~~ Simplify $\tan(\sin^{-1} x)$

angles numbers

arcsine = $\sin^{-1} x$

$\sin^{-1} x = \theta$ ← an angle.

$\sin\theta = x$

$\alpha^2 + x^2 = 1$

$\alpha = \sqrt{1-x^2}$

$\tan(\sin^{-1} x) = \tan\theta = \frac{x}{\sqrt{1-x^2}}$

H/W problem #72.

$\sin(2\cos^{-1} x)$

Hint: apply double angle formula.