Infinite Limits

Friday, February 13, 2015 1:30 PM

[Equation] means we can make [Equation] arbitarily large by taking *x* sufficiently close to *a*, but not equal to *a*.

"f(x) beomes large near a"

[Equation] means we can make [Equation] arbitarily large negative by taking *x* sufficiently close to *a*, but not equal to *a*.

"f(x) becomes a large negative number near a"

Example ① $\lim_{x\to 0} \frac{2}{x^2} =$

(Remember slide 1 $\frac{\text{positive number}}{\text{small positive}}$ = very large)

as $x \to 0$

x^2 is small positive so $\frac{2}{x^2} \approx \frac{2}{\text{small pos.}}$ = very large.

$$\lim_{x\to 0} \frac{2}{x^2} = +\infty$$

② $\lim_{x\to 3} \frac{1}{(x-3)^2} = +\infty$

as $x \to 3$

$x-3 \to 0$

i.e

$(x-3)$ is small (might be negative)

$(x-3)^2$ is small positive

so $\frac{1}{(x-3)^2} \approx \frac{1}{\text{small pos.}}$ = large.

[Equation] means *f(x)* can be made arbitrarily large by taking *x* close to a but bigger than *a.*

~~Variations~~

$$\lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty$$

$$\lim_{x\to a^-} f(x) = -\infty$$

Example ①

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty$$

x = small pos.

$\frac{1}{x} \approx$ very large

② $\lim_{x\to 3^-} \frac{x}{x-3} = -\infty$

$x \approx 3$ (but smaller than 3)

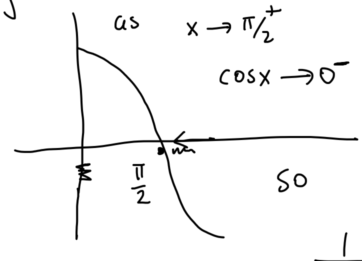
$x-3 \approx$ small negative

$$\frac{x}{x-3} \approx \frac{3}{\text{small negative}} \approx \text{very large negative}$$

Example

$$\lim_{x\to \frac{\pi}{2}^+} \sec x = \lim_{x\to\frac{\pi}{2}^+} \frac{1}{\cos x} = -\infty$$

$y = \cos x$



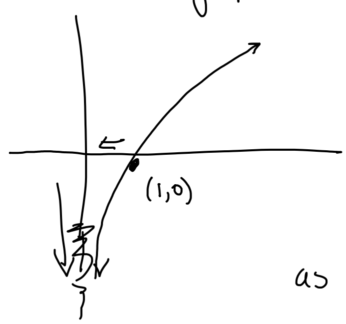
as $x \to \pi/2^+$

$\cos x \to 0^-$

so $\cos x$ is small & negative

$\frac{1}{\cos x} \approx$ is a large negative

Recall the graph of $y = \ln x$



as $x \to 0^+$

$\ln x \to -\infty$

Ex $\lim_{x\to -3^+} \ln(x+3) = -\infty$

as $x \to -3^+$ $(x \approx -3)$

$x+3 \approx 0$ and positive

$\ln(\text{small positive}) = \text{large negative}$