Symmetry

June 2, 2015 12:20 AM

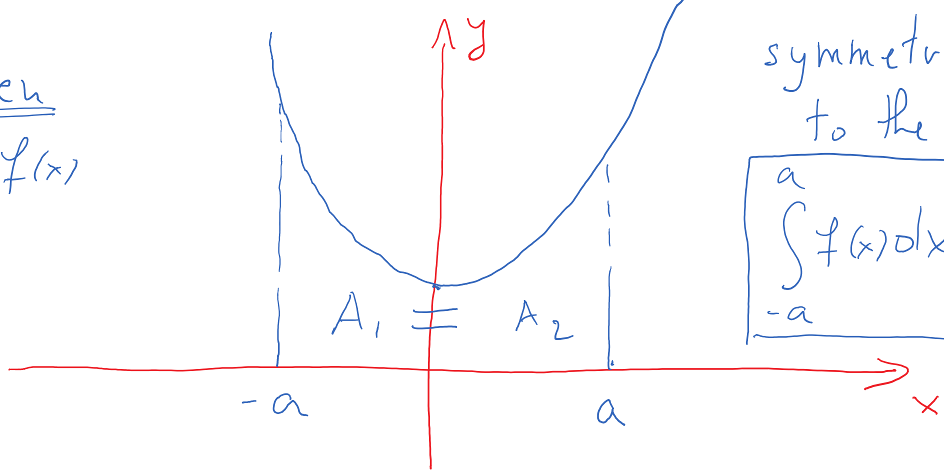
Symmetry

$f(x)$, $f(-x) = f(x) \Rightarrow f$ even function

$f(-x) = -f(x) \Rightarrow f$ odd function

f even

$f(-x) = f(x)$

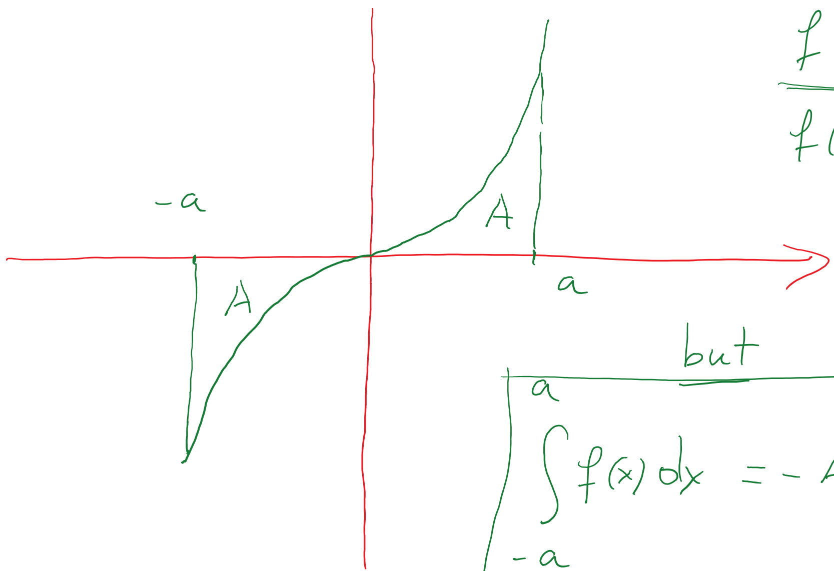


symmetric relative to the y axis.

$$\boxed{\int_{-a}^{a} f(x)\,dx = 2A_2 = 2\int_{0}^{a} f(x)\,dx}$$

f odd

$f(-x) = -f(x)$



but

$$\boxed{\int_{-a}^{a} f(x)\,dx = -A + A = \underline{\underline{0}}}$$

Examples.

1. $\displaystyle\int_{-2}^{2} (x^4+1)\,dx = 2\int_{0}^{2} (x^4+1)\,dx$

$f(x) = x^4 + 1$ <u>even</u>

$f(-x) = f(x)$

$$= 2 \cdot \frac{x^5}{5}\Big|_0^2 + 2x\Big|_0^2 = \frac{2 \cdot 32}{5} + 4 = \frac{64}{5} + 4 = \frac{84}{5}$$

$$\int_{-3}^{3} x^3 \sqrt{x^8 + 4x^2}\, dx = 0 \text{ by symmetry.}$$

DONE!

$f(x) = x^3\sqrt{x^8 + 4x^2}$ <u>odd function!</u>

$$f(-x) = (-x)^3 \sqrt{(-x)^8 + 4(-x)^2} = -x^3\sqrt{x^8 + 4x^2} = -f(x).$$

3. $$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x)\, dx = 0.$$

$$\underbrace{\underbrace{x^3}_{\text{odd}} + \underbrace{x^4}_{\text{even}}\underbrace{\tan x}_{\text{odd}}}_{\text{odd}}$$

odd . odd = even

even . even = even

even . odd = odd