Trigonometric Identities

Thursday, February 19, 2015 11:30 AM

Basic Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\cot\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

We already know these

Def^{n}: $\sin\theta = y/r \quad \cos\theta = x/r \quad \tan\theta = y/x$

So

$$\frac{\sin\theta}{\cos\theta} = \frac{(y/r)}{(x/r)} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x} = \tan\theta.$$

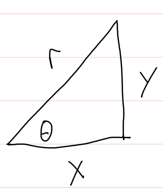
You should check for yourself that

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cot^2\theta + 1 = \sec^2\theta$$
$$1 + \tan^2\theta = \csc^2\theta$$

Check this for yourself, (divide the first by $\cos^2\theta$)



$(\div r^2)$

$$r^2 = y^2 + x^2$$
$$1 = \frac{y^2}{r^2} + \frac{x^2}{r^2}$$
$$1 = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

Remember $\sin\theta = y/r \quad \cos\theta = x/r$

$$1 = \sin^2\theta + \cos^2\theta \quad ✓$$

$(\sin^2\theta = (\sin\theta)^2$ etc by definition$)$

Divide the above by $\sin^2\theta$)

$$\frac{1}{\sin^2\theta} = 1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

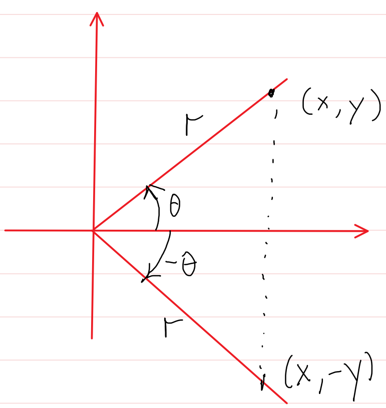
(Basic identity above)

$$\frac{1}{\sin^2\theta} = 1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 \qquad \left(\text{Basic identity above } \frac{\cos\theta}{\sin\theta} = \cot\theta\right)$$

$$\csc^2\theta = 1 + \cot^2\theta$$

Odd and Even identities

$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\tan(-\theta) = -\tan(\theta)$$



$$\sin(-\theta) = \frac{-y}{r} = -\left(\frac{y}{r}\right) = -\sin\theta$$

$$\cos(-\theta) = \frac{x}{r} = \cos\theta$$

↑ picture.

Check for yourself that $\tan(-\theta) = -\tan\theta$

Recall that a function $f(x)$ is <u>odd</u> if $f(-x) = -f(x)$

A function $g(x)$ is <u>even</u> if $g(-x) = g(x)$.

$\sin\theta$ & $\tan\theta$ are odd

$\cos\theta$ is even.

Angle sum formulae

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\alpha\cos\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

← Memorise!

Angle difference formulae

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\alpha\cos\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

← Chec this for yourself

Replase β with $-\beta$ in sum formula

$$\begin{aligned} \sin(\alpha-\beta) &= \sin(\alpha + (-\beta)) \\ \text{(sum formula)} \quad &= \sin\alpha\cos(-\beta) + \sin(-\beta)\cos(\alpha) \\ \text{(odd/even identities)} \quad &= \sin\alpha\cos(\beta) - \sin\beta\cos\alpha \quad \checkmark \end{aligned}$$

Double angle formula for sine

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

set $\alpha = \beta = \theta$ in sum formula.

$$\begin{aligned} \sin(2\theta) &= \sin(\theta + \theta) \\ &= \sin\theta\cos\theta + \sin\theta\cos\theta \\ &= 2\sin\theta\cos\theta \end{aligned}$$

Double angle formulae for cosine

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\cos(2\theta) = 1 - 2\sin^2\theta.$$

Check for yourself!

set $\alpha = \beta = \theta$ is sum formula for cosine

$\cos(2\theta) = \cos(\theta + \theta)$

$\cos(2\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ ☺

Remeber the pythogorean identity:

$\sin^2\theta + \cos^2\theta = 1$ plug in here:

$\sin^2\theta = 1 - \cos^2\theta$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = \cos^2\theta - (1 - \cos^2\theta)$

$\cos(2\theta) = \cos^2\theta - 1 + \cos^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$ ✓ ☺