Solving Trigonometric Equations

Saturday, February 21, 2015 10:36 PM

Example Find all x in $[0, 2\pi]$ that satisfy $3\cot^2 x = 1$

$0 \le x \le 2\pi$

Start with $3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm \frac{1}{\sqrt{3}}$

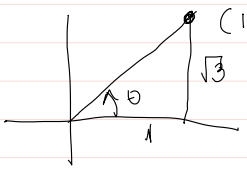
Replace x with θ $\cot\theta = \pm \frac{1}{\sqrt{3}}$

Recall $\cot\theta = \frac{x}{y}$.

Look for solutions quadrant by quadrant

Quad I $x, y \ge 0$

so $\cot\theta = \frac{x}{y} \ge 0$ $\cot\theta = \frac{1}{\sqrt{3}}$



special $\triangle$

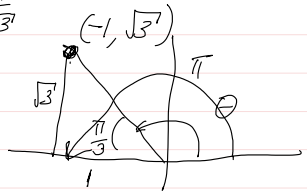
$\theta = \frac{\pi}{3}$

Quad II $x \le 0$ $y \ge 0$

so $\cot\theta = \frac{x}{y} \le 0$ negative

$\cot\theta = -\frac{1}{\sqrt{3}}$

Special $\triangle$



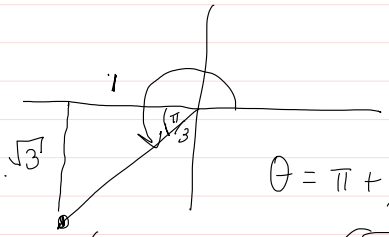
$\theta = \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Quad III $x \le 0, y \le 0$ $\cot\theta = \frac{x}{y} = \frac{\text{negative}}{\text{negative}} \ge 0$

$\cot\theta = \frac{1}{\sqrt{3}}$

Special $\triangle$



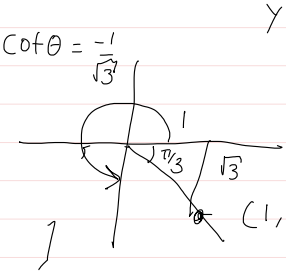
$\theta = \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

Quad IV $x \ge 0$ $y \le 0$

$\cot\theta = \frac{x}{y} = \frac{\text{pos.}}{\text{neg}} \le 0$

$\cot\theta = -\frac{1}{\sqrt{3}}$



1 rev.

$\theta = 2\pi - \frac{\pi}{3}$

$= \frac{6\pi}{3} - \frac{\pi}{3}$

$= \frac{5\pi}{3}$

Answer : $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Example Find all solutions to $2\cos x + \sin 2x = 0$ in $\mathbb{C}$, (i.e $0 \le x \le 2\pi$)

$0 = 2\cos x + \sin 2x$

$0 = 2\cos x + 2\sin x \cos x$ (double angle formula)

$0 = 2\cos x(1 + \sin x)$

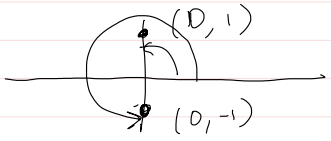
$\cos x = 0$ or $1 + \sin x = 0$

$\sin x = -1$

2 possibilities!

Case $\cos x = 0$ replace $x \to \theta$ $\cos\theta = 0$

$\frac{x}{r} = \cos\theta = 0$ so $x = 0$



$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Case $\sin x = -1$ replace $x \to \theta$ $\sin\theta = -1$

$\frac{y}{r} = \sin\theta = -1$

$|y|$ = absolute value of y

$|x|, |y| \le r$ from picture

so $\frac{y}{r} = -1$ means $|y| = r$

can make $y = -1$, $r = 1$

this means $x = 0$!

Final Answer

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ← twice