Proving Identities

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Example Show that

$$\sin x \sin 2x + \cos x \cos 2x = \cos x$$

$$\begin{aligned}
\text{LHS} &= \sin x \sin 2x + \cos x \cos 2x \\
&= \sin x \,(2\sin x \cos x) + \cos x \,(\cos^2 x - \sin^2 x) \\
&= 2\sin^2 x \cos x + \cos^3 x - \cos x \sin^2 x \\
&= \sin^2 x \cos x + \cos^3 x \\
&= \cos x \,(\sin^2 x + \cos^2 x) \\
&= \cos x \,(1) \\
&= \cos x = \text{RHS}
\end{aligned}$$

Example 2 Show that

$$\frac{\sin\theta}{1-\cos\theta} = \csc\theta + \cot\theta$$

$$\begin{aligned}
\text{RHS} &= \csc\theta + \cot\theta \\
&= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \\
&= \frac{1+\cos\theta}{\sin\theta}
\end{aligned}$$

Trick! multiply by conjugate of $1-\cos\theta$ i.e $1+\cos\theta$

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta}{1-\cos\theta} \\
&= \frac{\sin\theta}{1-\cos\theta} \cdot 1 \\
&= \frac{\sin\theta}{(1-\cos\theta)} \cdot \frac{(1+\cos\theta)}{(1+\cos\theta)} \\
&= \frac{\sin\theta\,(1+\cos\theta)}{1^2-\cos^2\theta} \\
&= \frac{\cancel{\sin\theta}\,(1+\cos\theta)}{\sin^{\cancel{2}}\theta}
\end{aligned}$$

$\sin^2\theta$

$$= \frac{1+\cos\theta}{\sin\theta} = RHS$$