Laws of Exponents If a and b are positive numbers and x and y are any real numbers, then

1. $b^{x+y} = b^x b^y$ 2. $b^{x-y} = \frac{b^x}{b^y}$ 3. $(b^x)^y = b^{xy}$ 4. $(ab)^x = a^x b^x$

Example Simplify $\frac{(6y^3)^4}{2y^3}$

$\frac{(6y^3)^4}{2y^3} = \frac{6^4 (y^3)^4}{2y^3}$ (use 4)

$= \frac{6^4 y^{12}}{2y^3}$ (use 3)

No calculator

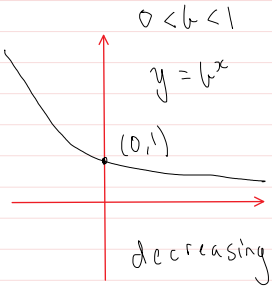
$= \frac{(3\cdot 2)^4 y^{12}}{2y^3}$ $6 = 3\times 2$

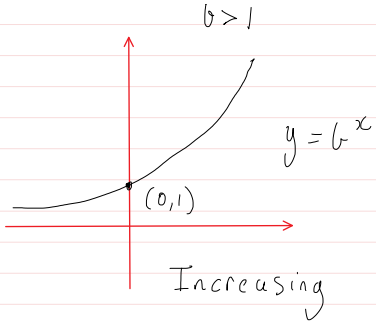
$= \frac{3^4 2^4 y^{12}}{2y^3}$ (use 3)

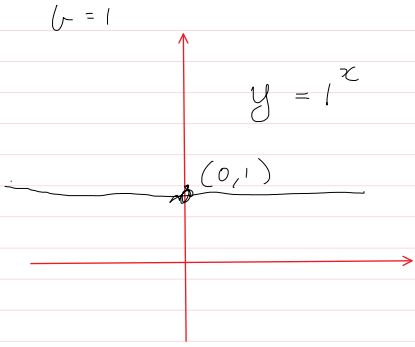
$= 3^4 \cdot 2^3 y^9$ (use 2)

$= (81)(8) y^9$

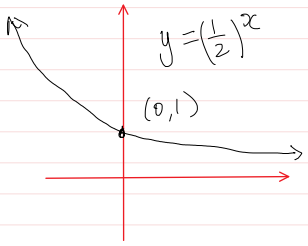
Suppose that $b > 0$ is some fixed number. A function of the form $f(x) = b^x$ is called an *exponential function.*







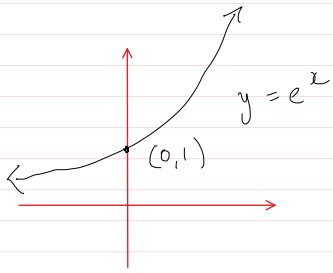
Example ① Sketch $y = \left(\frac{1}{2}\right)^x$ $0 < b = \frac{1}{2} < 1$



② Sketch $y = e^x$ e = Euler's constant $\approx 2.7128\ldots$

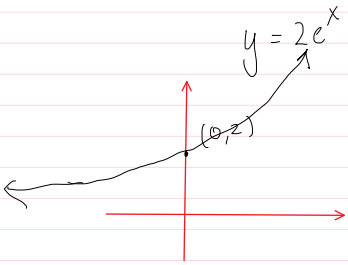
approximately equal.

$1 < e$

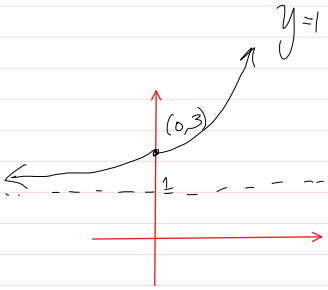


③ Sketch $y = 1 + 2e^x$

$y = 2e^x$ (Stretch by 2)

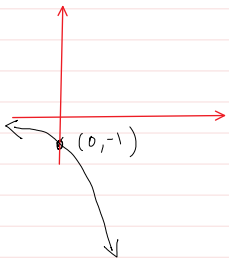


$y = 1 + 2e^x$ Shift up 1

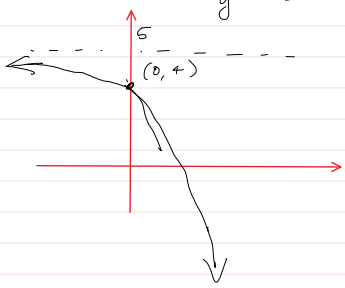


Example Sketch $y = 5 - e^x$

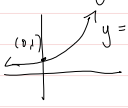
$y = -e^x$ reflect in x-axis



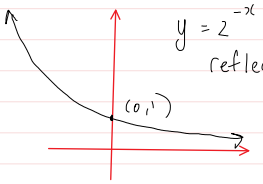
$y = 5 - e^x$ Shift up 5



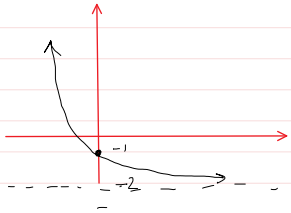
Example Sketch $y = 2^{-x} - 2$



$y = 2^{-x}$ reflect in y-axis

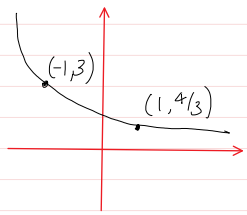


$y = 2^{-x} - 2$ shift down 2



Example Find the exponential function $y = Cb^x$

with graph:



$f(x) = Cb^x$

what are C and b?

$\frac{4}{3} = f(1) = Cb$ $\quad\Rightarrow\quad \frac{4}{3} = Cb$ (1)

$3 = f(-1) = Cb^{-1}$ $\quad\Rightarrow\quad 3 = \frac{C}{b}$ (2)

From (2) $3b = C$

Subs into (1): $\frac{4}{3} = (3b)\,b$

$\frac{4}{3} = 3b^2$

$\frac{2^2}{3^2} = b^2$ $\quad b = \pm\frac{2}{3}$

but $f(x) = Cb^x$ so $b > 0$

$b = \frac{2}{3}$

$C = 3b = 3\left(\frac{2}{3}\right) = 2$

Answer

$f(x) = 2\left(\frac{2}{3}\right)^x$