Getting ready for week 2

Wednesday, February 11, 2015 10:20 AM

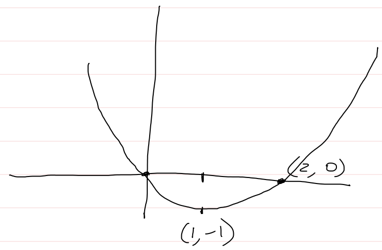
Sketch $y = x^2 - 2x = f(x)$

x-intercepts. Set $y = 0$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0$ or $x = 2$



Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

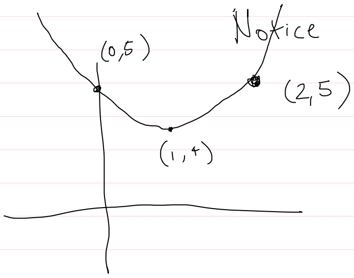
$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

<u>Example</u> Sketch $y = x^2 - 2x + 5$

$f(x) = x^2 - 2x$ Sketch $y = f(x) + 5$

Shift up 5 units

Notice we sketch $y = f(x)$ in the previous example



Example $\underbrace{y = (x-1)^2 - 2(x-1)}_{\text{Sketch.}}$

Consider $f(x) = x^2 - 2x$.

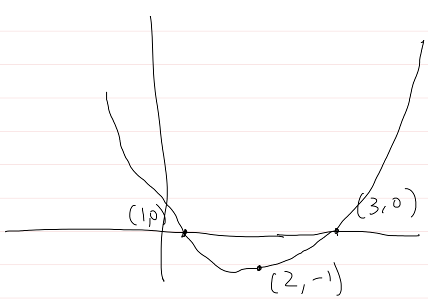
We want to sketch $y = f(x-1)$

Shift to the right 1 unit

$(0,0) \longrightarrow (1,0)$

$(2,0) \longrightarrow (3,0)$

$(1,-1) \longrightarrow (2,-1)$



Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

- $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
- $y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c
- $y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c
- $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

Original parabola.

Example sketch $y = 2(x^2 - 2x)$ $\qquad f(x) = x^2 - 2x$

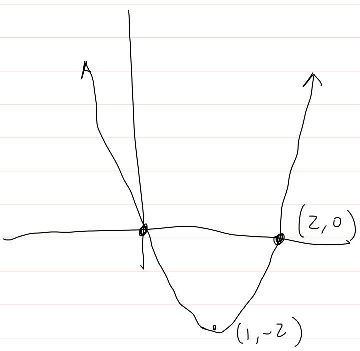
sketch $y = 2f(x)$

Stretch vertically by 2. (multiply y-coords by 2)

$(0,0) \longrightarrow (0,0)$

$(2,0) \longrightarrow (2,0)$

$(1,-1) \longrightarrow (1,-2)$



Example $y = (2x)^2 - 2(2x)$ (sketch)

$y = f(x) = x^2 - 2x$ original

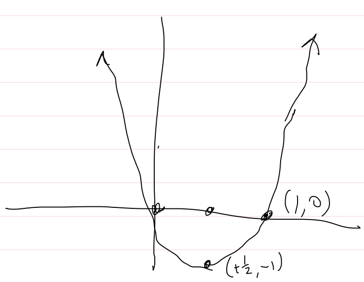
We need to now sketch $y = f(2x)$

Shrink horizontally by a factor of 2. (divide x-coords by 2)

$(0,0) \longrightarrow (0,0)$

$(2,0) \longrightarrow (1,0)$

$(1,-1) \longrightarrow (\frac{1}{2},-1)$



Example Sketch $\qquad f(x) = x^2 - 2x$ (original)

$y = -(x^2 - 2x)$ $\quad$ & $\quad$ $y = x^2 + 2x$

$f(x) = x^2 - 2x$ (original)

Example Sketch

$y = -(x^2 - 2x)$ & $y = x^2 + 2x$

$y = -f(x)$ $\qquad$ $= (-x)^2 - 2(-x)$

$y = f(-x)$

X-axis reflection $\qquad$ reflect in y-axis.

mult y by −1

$(0,0) \to (0,0)$

$(2,0) \to (2,0)$

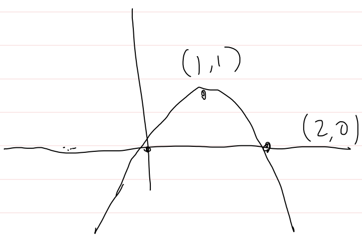
$(1,-1) \to (1,1)$

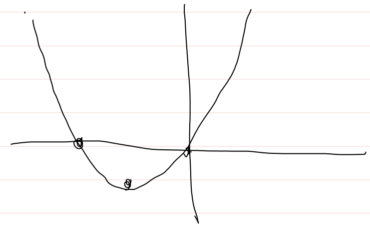
mult x by −1

$(0,0) \to (0,0)$

$(2,0) \to (-2,0)$

$(1,-1) \to (-1,-1)$





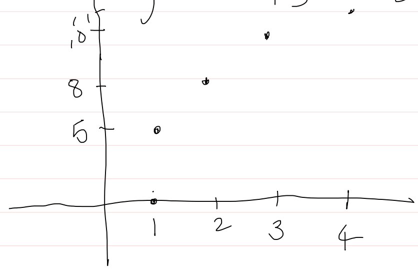
A function is said to be one-to-one if it never takes the same value twice. In other words if $f(x)$ is one-to-one then $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Example Consider a function f with domain equal to just four numbers 1, 2, 3, 4

Specify f via a table of values

x	f(x)
1	5
2	8
3	10
4	11

Range is 5, 8, 10, 11



This function is 1-to-1 as it never takes the same value twice

Example 2 Consider the function g with domain 1, 2, 3, 4

x	g(x)
1	8
2	8

g(x) is not 1-to-1 as

1	8
2	8
3	10
4	10

$g(x)$ is not 1-to-1 as $g(1) = 8 = g(2)$.

Suppose that $f(x)$ is a one-to-one function with domain D and range R. Then one can construct the inverse function $f^{-1}(x)$ of $f(x)$. The inverse function has domain R and range D. It is defined by

$a = f^{-1}(b)$ if and only if $f(a) = b$.

Warning: $f^{-1}(x) \neq \dfrac{1}{f(x)}$.

Example Recall from example 1 above the function $f(x)$:

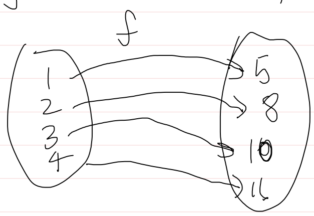
x	$f(x)$
1	5
2	8
3	10
4	11

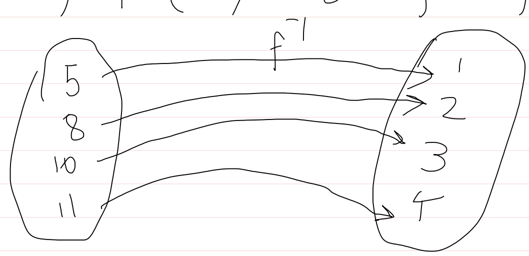
$f(x)$ is 1-to-1 with domain $\{1, 2, 3, 4\}$ range $\{5, 8, 10, 11\}$

We can form the inverse function.

$f^{-1}(x)$ has domain $\{5, 8, 10, 11\}$ range $\{1, 2, 3, 4\}$

$f^{-1}(5) = 1$, $f^{-1}(8) = 2$; $f^{-1}(10) = 3$ $f^{-1}(11) = 4$





Remember g from above

Remember g from above

x	$g(x)$
1	5
2	5
3	8
4	8

g^{-1} does not make sense

why?

$$g^{-1}(5) = \begin{cases} 1 \\ 2 \end{cases}$$

which one?

A function can only take one value!