1. The exponential function $f(x) = Ca^x$ whose graph passes through points $(1, 4)$ and $(2, 32)$ is
   A. $f(x) = 4^x$
   B. $f(x) = 5^x$
   C. $f(x) = \frac{1}{2}\ 8^x$
   D. $32^{x/2}$
   E. none of the above

2. How many solutions does the equation $\sin(2x) = \cos x$ have in the interval $[0, 2\pi]$?
   A. none
   B. one
   C. two
   D. three
   E. more than three

3. Which of the following functions has an inverse:
   A. f(x)=3
   B. $g(x) = x^2$
   C. $h(x) = x^2 + 1$
   D. $k(x) = e^x$
   E. none of the above

4. If $f(x) = \ln(x + 5)$ then $f^{-1}(x) =$
   A. e^{x-5}
   B. $e^x - 5$
   C. $e^{x-\ln 5}$
   D. $\frac{1}{\ln(x-\frac{1}{5})}$
   E. none of the above

5. (Remember no calculators or electronic aids may be used in the exam)
   $\log_3 \frac{1}{27} + \log_3 30 - \log_3 10 =$
   A. -3
   B. 3
   C. -2
   D. 2
   E. none of the above

6. $\tan(\arctan 8) =$
   A. -8
   B. 8
   C. $\pi - 8$
   D. $\frac{\pi}{2} - 8$
   E. none of the above

7. $\cos(2\tan^{-1} x) =$
   A. $\frac{1+x^2}{1-x^2}$
   B. $\frac{1-x^2}{1+x^2}$
   C. $\frac{x}{1+x^2}$
   D. $\frac{x}{1-x^2}$
   E. none of the above
8. $\lim_{x\to 4^+} \ln(x^2-16) =$
   A. 0
   B. 1
   C. $-\infty$
   D. ∞
   E. none of the above
9. $\lim_{h\to 0} \frac{(3h+1)^2-1}{h} =$
   A. 0
   B. 6
   C. 9
   D. 15
   E. none of the above
10. $\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2} =$
   A. 0
   B. $-\frac{1}{4}$
   C. $-\frac{1}{2}$
   D. -1
   E. none of the above
11. $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} =$
   A. -1
   B. 0
   C. 1
   D. 2
   E. none of the above
12. $\lim_{x\to\infty} \arctan(e^{2x}) =$
   A. $\frac{\pi}{2}$
   B. 0
   C. ∞
   D. $-\infty$
   E. none of the above

13. $\lim_{x\to-\infty} \frac{x^{1/3}+x^2}{1+x+2x^2} =$
    A. $-\frac{1}{2}$
    B. 0
    C. $\frac{1}{2}$
    D. 1
    E. none of the above

14. For the graph of the function $y = \frac{2x^2+x-1}{x^2+x-2}$ which of the following statements is true ?
    A. This graph has two vertical asymptotes and one horizontal asymptote
    B. x=3 is a vertical asymptote
    C. y=3 and y=0 are horizontal asymptotes
    D. there are no vertical asymptotes
    E. statements A,B,C,D are false

15. Equality $\lim_{h\to 0} \frac{\sin(\frac{\pi}{2}+h)-1}{h} = f'(a)$ holds for
    A. $f(x) = \sin x,\ a = 0$
    B. $f(x) = \sin x,\ a = \frac{\pi}{2}$
    C. $f(x) = \sin(\frac{\pi}{2}+x),\ a = \frac{\pi}{2}$
    D. $f(x) = \sin(\frac{\pi}{2}+x) - 1,\ a = 0$
    E. none of the above

16. $\frac{d}{dx}e^5 =$
    A. 0
    B. e^5
    C. $5e^4$
    D. $\ln 5$
    E. none of the above

17. The slope of the tangent line to the graph of $y = \sin x - \cos x$ at $(\pi, 1)$ is
    A. 2
    B. 1
    C. 0
    D. -1
    E. none of the above

18. If $f(x) = x^2 \cos x$, then $f''(x) =$
    A. $-2\cos x$
    B. $-2x\sin x$
    C. $-2x\cos x - 2\sin x$
    D. $-4x\sin x + (-x^2+2)\cos x$
    E. none of the above

19. For $f(x) = 5^x$ its 8-th derivative $f^{(8)}(x)$ is
    A. 5^x
    B. $(\ln 5)^7 5^x$
    C. $(\ln 5)^8 5^x$
    D. $(\ln 5)^9 5^x$
    E. none of the above

The following multiple choice problems were from the 2014 final that are relevant to this midterm.

20. $\frac{d}{dx}\cos(e^x + x) =$
    A. $\sin(e^x + x)$
    B. $-\sin(e^x + x)$
    C. $(e^x + 1)\sin(e^x + x)$
    D. $-(e^x + 1)\sin(e^x + x)$
    E. none of the above

21. $\lim_{x\to -3} \frac{x+3}{x^2-9} =$
    A. $\frac{1}{6}$
    B. $-\frac{1}{6}$
    C. ∞
    D. 0
    E. none of the above

22. An equation of the tangent line to $y = 3e^{x-1}$ at $(1, 3)$ is
    A. $y = 3x$
    B. $y = \frac{3}{e}x$
    C. $y = 3x - 1$
    D. $y = 3x - e$
    E. none of the above

23. If $g(x) = \sin(f(x) + \pi)$, $f(\pi) = 0$, and $f'(\pi) = 2$, then $g'(\pi) =$
    A. π
    B. 2
    C. 0
    D. -2
    E. none of the above

24. If $xe^y = x - y$ then the value of $\frac{dy}{dx}$ when $x = 1$ and $y = 0$ is
    A. 0
    B. $\frac{1}{2}$
    C. 1
    D. 2
    E. none of the above

25. If $f(x) = \ln(\ln(x^2 + 1))$ then $f'(1) =$
    A. $\ln(\ln 2)$
    B. $\frac{2}{\ln 2}$
    C. $\frac{1}{\ln 2}$
    D. $\frac{1}{2\ln 2}$
    E. none of the above

26. If $y = (\sin x)^{2x}$ then $y' =$

A. $2x(\sin x)^{2x-1}$

B. $(\cos x)^{2x}$

C. $(\sin x)^{2x}(2\ln(\sin x) + 2x\cot x)$

D. $(\sin x)^{2x}(2\ln(\sin x) - \frac{2x}{\sin x})$

E. none of the above

27. For the function $f(x) = 2x^3 - 3x^2 - 36x$ which of the following statements is true ?

A. 0 is the only critical number

B. there are three critical numbers

C. there are no critical numbers

D. -2 and 3 are the critical numbers

E. statements A,B,C,D are false

The following are short answer questions from the 2014 midterm.

In the questions that follow you are required to show your work. Questions without sufficient justification will not receive credit.

28. Let

$$f(x) = \begin{cases} \frac{x^2+x-6}{|x-2|} & \text{if } x \neq 2 \\ 5 & if\ x = 2 \end{cases}$$

(i) Find $\lim_{x\to 2-} f(x)$

(ii) Find $\lim_{x\to 2+} f(x)$

(iii) Find $\lim_{x\to 2} f(x)$

(iv) Is $f(x)$ continuous at $x = 2$? Why ?

(v) Is $f(x)$ continuous at $x = 0$? Why ?

29. Use the Intermediate Value Theorem to show that the equation

$$4^x = 5 - 3x$$

has a solution in the interval $(0, 1)$.

30. For

$$f(x) = \cos(x^2 + \arctan(e^x \sin x))$$

find $f'(x)$.

31. If $F(x) = f(g(x))$ where $f(\pi) = 8$, $f'(\pi) = 4$, $f'(6) = 3$, $g(6) = \pi$, and $g'(6) = 7$, find $F'(6)$.

32. (a) If $x^2 - xy + y^2 = 3$, use implicit differentiation to find $\frac{dy}{dx}$ when $x = -1$, $y = 1$.

(b) Use your answer in part (a) to find an equation of the tangent line to the curve $x^2 - xy + y^2 = 3$ at $(-1, 1)$.

The following are questions from the 2014 final relevant to this midterm.

33. A cylindrical tank with radius 4 m is being filled with water at a rate of 3 m^3/min. How fast if the height of the water increasing ?

34. Find the absolute maximum and absolute minimum values of $f(x) = x\sqrt{4 - x^2}$ on $[-1, 2]$.

35. If 12 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.