1. Which (if any) of the following functions has an inverse?

   (A) $f(x) = 0$ (B) $f(x) = x^3 + 2$ (C) $f(x) = 1$ (D) $y(x) = e^{x^4} - 5$ (E) none of (A) to (D)

2. $\lim_{x \to \pi -} \ln(\sin x) =$

(A) 0 (B) ∞ (C) $-\infty$ (D) 1 (E) none of (A) to (D)

3. If $\lim_{x \to 2} \dfrac{f(x) - 1}{x - 2} = \pi$, then $\lim_{x \to 2} f(x)$ is equal to

(A) 0 (B) 2 (C) 1 (D) π (E) ∞

4. $\lim_{x\to\infty} \dfrac{-2x^3+5x-1}{x^3-1} =$

(A) 1 (B) ∞ (C) 0 (D) $-\infty$ (E) -2

5. $\lim_{x \to -\infty} (0.1)^x =$

(A) 1 (B) -1 (C) 0 (D) $-\infty$ (E) ∞

6. $\sin(\arctan x) =$

(A) $\dfrac{x}{\sqrt{1-x^2}}$ (B) $\dfrac{1}{\sqrt{1+x^2}}$ (C) $\dfrac{1}{\sqrt{1-x^2}}$ (D) $\dfrac{x}{\sqrt{1+x^2}}$ (E) none of (A) to (D)

7. The domain of the function $f(x) = \ln(\ln(x^2 + 1))$ is

(A) $(0, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$ (C) $(1, \infty)$ (D) $(1, e)$ (E) $(1/e, e)$

8. $\cos(2\arcsin(3/5)) =$

(A)$7/25$ (B) $4/5$ (C) $-4/5$ (D) $-7/25$ (E) none of (A) to (D)

9. If $F(\theta) = \sin\theta$, then $\dfrac{d^5}{dx^5}F(\theta) =$

(A) $\sin\theta$ (B) $-\cos\theta$ (C) $-\sin\theta$ (D) $\cos\theta$ (E) none of (A) to (D)

10. $\lim_{x \to -\infty} \arctan x =$

(A) $\frac{\pi}{2}$ (B) -1 (C) $-\frac{\pi}{2}$ (D) 1 (E) does not exist

11. $\dfrac{d}{dx}e^{\sqrt{x^2+1}} =$

(A) $\sqrt{x^2+1}e^{\sqrt{x^2+1}-1}$ (B) $e^{\sqrt{x^2+1}-1}$ (C) $\dfrac{1}{2\sqrt{x^2+1}}e^{\sqrt{x^2+1}}$

(D) $\dfrac{x}{\sqrt{x^2+1}}e^{\sqrt{x^2+1}}$ (E) none of (A) to (D)

12. $\lim_{t \to -\infty} \dfrac{\sqrt{9t^2 + t - 2}}{t - 3} =$

(A) $-\infty$ (B) ∞ (C) 0 (D) 3 (E) -3

13. If $x^2 + y^2 = 25$, then $\dfrac{dy}{dx} =$

(A) 1 (B) $-\dfrac{x}{y}$ (C) $-\dfrac{y}{x}$ (D) $-2x$ (E) none of (A) to (D)

14. If $f(x)$ is continuous on $[1, 9]$, $f(1) = -2$ and $f(9) = 1$, which of the following statements is necessarily true?

(A) there is at most one x in the interval $[1, 9]$ such that $f(x) = 0$
(B) there is at most one x in the interval $[1, 9]$ such that $f'(x) = 0$
(C) the graph of $f(x)$ has a vertical asymptote between 1 and 9
(D) there is at least one x in the interval $[1, 9]$ such that $f(x) = 0$
(E) none of (A) to (D)

15. If $g(x) = \ln(x+3)$, then its inverse function is $g^{-1}(x) =$

(A) $e^{(x+3)}$ (B) $e^x - 3$ (C) $e^x + 3$ (D) $e^{(x-3)}$ (E) none of (A) to (D)

16. $\lim_{x \to -2+} \log_2 \left(\frac{1}{6 - x} \right) =$

(A) -3 (B) $-\infty$ (C) 3 (D) ∞ (E) none of (A) to (D)

17. If $F(x) = f(4f(x)), \quad f(0) = 0$ and $f'(0) = 5$, evaluate $F'(0)$. **[8 marks]**

18. Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$$

[8 marks]

19. Use the Intermediate Value Theorem to show that the equation $x^{\frac{1}{3}} = 1 - x$ has a root in the interval $(0, 1)$. **[8 marks]**

20. If $y + x \sin y = x^2$, find $\dfrac{dy}{dx}$ when $x = 0$. **[8 marks]**

21. Calculate the derivative of the given function:

$y = x \arcsin x + \sqrt{1 - x^2}$ **[8 marks]**

22. Find an equation of the tangent line to the graph of $y = \ln x$ which passes through the point $(0, 0)$. Note: $(\ln x)' = 1/x$. **[8 marks]**