Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENTS OF APPLIED MATHEMATICS AND MATHEMATICS

Calculus 1000A Final Examination

Monday, December 21, 2015 **Code 111** 9:00 a.m. – 12:00 p.m.

INSTRUCTIONS

1. The first part of the exam (Part A) is MULTIPLE CHOICE. This part is to be answered on the SCANTRON answer sheet. As well, make sure that you circle your selected answer in the examination booklet. The answer you give on the scantron sheet is taken as being your intended choice in the event that the answer in the examination booklet is not the same. The second part (Part B) has questions to be answered in the space provided. Be sure to answer each part of a question in the space provided for that part of the question. INDICATE YOUR ANSWER CLEARLY.
2. SHOW ALL OF YOUR WORK FOR PROBLEMS IN PART B. All results must be justified unless you are instructed otherwise. Unjustified answers will receive little or no credit.
3. Print your name and your instructor's name on the SCANTRON answer sheet. Sign the answer sheet, and mark your student number, section and exam code on the scantron sheet. USE AN HB PENCIL to mark your answers on the scantron answer sheet.
4. DO NOT UNSTAPLE THE BOOKLET.
5. Questions start on Page 1 and continue to Page 16. Questions are printed on both sides of the paper. Should you require extra space for any answer, Pages 17 and 18 are provided for this purpose. BE SURE YOUR BOOKLET IS COMPLETE.
6. CALCULATORS ARE NOT PERMITTED.
7. Leave your scantron sheet on top of your examination booklet when turning them in.
8. Total Marks = 100.
9. Fill in the top of this page. CIRCLE YOUR SECTION NUMBER IN THE LIST BELOW.

001	N. Kiriushcheva (8:30 - 9:30am)	006	J. Turnbull
002	C. Bibby	007	P. Gupta (7 - 9pm)
003	J. Adamus	008	P. Gupta (2:30 - 3:30pm)
004	N. Kiriushcheva (1:30 - 2:30pm)	009	P. Yu
005	A. MacIsaac	011	I. Kobyzev
570	D. Meredith (King's)		

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
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TOTAL	

PART A (50 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

2 marks A1. The range of the function $f(x) = e^{|\sin x|}$ is

A: $(-\infty, \infty)$	B: $(0, \infty)$	C: $[0, 1]$	D: $[1, e]$	E: $[1, \infty)$

2 marks A2. Which of the following is an even function?

A: $f(x) = x \cos x$	B: $f(x) = x^2 \ln x$	C: $f(x) = \dfrac{\sin x}{\sqrt[3]{x}}$	D: $f(x) = \sqrt{x-2}$	E: $f(x) = x^2 \tan x$

2 marks A3. From symmetry, the integral $\displaystyle\int_{-2}^{2} e^{-x^2} \sin x \, dx$ is equal to

A: $-\cos 2 \cdot (e^4 - e^{-4})$	B: $-2e^4 \cos 2$	C: 0	D: $e^4 \sin 2$	E: $\cos 2 \cdot (e^4 + e^{-4})$

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2 marks A4. If the derivative of a function f is $f'(x) = 3x(x-1)^4(2-x)^3$, then f is increasing on the interval

A: $(-\infty, 0)$	B: $(-2, -1)$	C: $(-2, 0)$	D: $(0, 2)$	E: $(2, \infty)$

2 marks A5. $\dfrac{d}{dx}\arcsin(2\sqrt{x}) =$

A: $\dfrac{\sin(2\sqrt{x})}{\sqrt{x}}$	B: $\dfrac{\arccos(2\sqrt{x})}{\sqrt{x}}$	C: 0	D: $\dfrac{\sqrt{x}}{\sqrt{1-4x}}$	E: $\dfrac{1}{\sqrt{x}} \cdot \dfrac{1}{\sqrt{1-4x}}$

2 marks A6. If $\cos y = \ln(\sqrt{x} + y)$, then $\dfrac{dy}{dx}$ at the point $(e^2, 0)$ is

A: $-2e^2$	B: $\dfrac{-1}{2e}$	C: 0	D: $\sin(e^2)$	E: $\dfrac{1}{2e^2}$

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2 marks A7. $\lim_{x \to -\infty} \dfrac{\ln(1+e^{-x})}{x} =$

A: $-\infty$	B: -1	C: 0	D: 1	E: ∞

2 marks A8. If $f(x) = \sin(3\sin(4x))$, then $f'(0) =$

A: 0	B: 1	C: 3	D: 4	E: 12

2 marks A9. If $f(x) = (2x)^{\cos x}$, then $f'(x) =$

A: $2 \cdot (2x)^{-\sin x}$	B: $-2\sin x$	C: $(2x)^{\cos x}\left(\dfrac{\cos x}{x} - \sin x \cdot \ln(2x)\right)$
D: $(2x)^{\cos x}\left(\cos x \cdot \ln(2x) - \dfrac{\sin x}{x}\right)$		E: $2^{-\sin x}$

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2 marks A10. The critical numbers of the function $f(x) = \sqrt{9 - x^2}$ are

A: -3, 0 and 3	B: -3 and 3	C: 0	D: -9 and 9	E: -9, 0 and 9

2 marks A11. How many inflection points does the function $f(x)$ have if $f''(x) = x^2 e^x (x+2)(x+6)$?

A: none	B: one	C: two	D: three	E: more than three

2 marks A12. $\lim_{n\to\infty} \left(\sum_{k=2}^{n} \left(\frac{1}{k+1} - \frac{1}{k+2} \right) \right) =$

A: $-\infty$	B: -1	C: 0	D: $\frac{1}{3}$	E: 1

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2 marks A13. $\lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{3}{n}\cdot\left(\frac{3i}{n}\cos\left(\frac{9i^2}{n^2}\right)\right)\right] =$

A: $\int_0^1 3(\cos t)^2 dt$	B: $\int_0^1 3t\cos(t^2)dt$	C: $\int_0^3 3(\cos t)^2 dt$	D: $\int_0^3 t\cos(t^2)dt$	E: ∞

2 marks A14. If $f(x) = \sqrt{9-x^2}$, then the area under the graph of f over the interval $[0,3]$ is

A: 1	B: 3	C: π	D: $\frac{9\pi}{4}$	E: 9

2 marks A15. $\int_1^e \frac{\ln(x^2)}{x}\,dx =$

A: 1	B: $e^{-3}-1$	C: $\frac{2}{e}$	D: $\frac{1}{2}$	E: $\frac{e-1}{2}$

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2 marks A16. $\lim_{x\to 0} \dfrac{\int_0^x (\sin t)^2 dt}{x^2} =$

A: $-\infty$	B: -1	C: 0	D: 1	E: 2

2 marks A17. $\dfrac{d}{dx}\displaystyle\int_{\frac{\pi}{2}}^{x^2} \sin t \; dt =$

A: 0	B: $\sin(x^2) - 1$	C: $\cos(x^2)$	D: $\frac{\pi}{2}\cos(x^2)$	E: $2x\sin(x^2)$

2 marks A18. If f is continuous, then $\displaystyle\int_0^4 \frac{f(\sqrt{4-x})}{\sqrt{4-x}} dx =$

A: $\displaystyle\int_0^4 2f(u)du$	B: $\displaystyle\int_0^2 2f(u)du$	C: $\displaystyle\int_0^2 -2f(u)du$	D: $\displaystyle\int_0^2 f(u)du$	E: $\displaystyle\int_0^4 f(u)du$

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2 marks A19. If $\int_{x^3}^{x^2} f(t)dt = 2$ and $\int_{0}^{x^3} f(t)dt = 3$, then $\int_{x^2}^{0} f(t)dt =$

A: -5	B: -1	C: 0	D: 1	E: 5

2 marks A20. If $f'(x) = \dfrac{1}{1+x^2} + \pi$ and $f(0) = 0$, then $f(1) =$

A: 1	B: $\dfrac{1}{2} + \pi$	C: $\dfrac{5\pi}{4}$	D: $\dfrac{\pi}{4}$	E: 4π

2 marks A21. $\int_{0}^{1} \sqrt[3]{8-7x}\, dx =$

A: -1	B: $\dfrac{28}{15}$	C: 1	D: $\dfrac{45}{28}$	E: $\dfrac{9}{28}$

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2 marks A22. If $f''(x) = \sin(2x)$, $f(0) = 1$ and $f(\frac{\pi}{4}) = 0$, then $f(x) =$

A: $-\frac{1}{4}\sin(2x)$	B: $-\frac{1}{4}\sin(2x) + 1$	C: $-\frac{1}{4}\sin(2x) - \frac{3x}{\pi}$	D: $-\frac{1}{4}\sin(2x) - \frac{3x}{\pi} + 1$
E: $-\sin(2x)$			

2 marks A23. $\displaystyle\int \frac{x^2 - 1}{x^4}\, dx =$

A: $\frac{x^2}{\arctan x} + C$	B: $\frac{5(x^3 - x)}{3x^5} + C$	C: $\frac{1}{3x^3} - \frac{1}{x} + C$	D: $\frac{1 + 3x^3}{x^3} + C$
E: none of the above			

2 marks A24. $\displaystyle\int \cos x\sqrt{\sin x}\, dx =$

A: $-\sin x\sqrt{\cos x} + C$	B: $-\frac{\sin x}{2\sqrt{\cos x}} + C$	C: $-\frac{(\sin x)^2}{2\sqrt{\cos x}} + C$	D: $\frac{2}{3}\sqrt{(\sin x)^3} + C$
E: none of the above			

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2 marks A25. If f has a continuous derivative on the interval $[-5, 10]$, $f'(-5) = -1$ and $f'(10) = 1$, which of the following statements is necessarily true?

A: f is increasing on $[-5, 10]$
B: $f(c) = 0$ for some c in $(-5, 10)$
C: f is concave up on the interval $[-5, 10]$
D: The graph of f has a vertical asymptote between -5 and 10
E: The graph of f has a horizontal tangent between -5 and 10

PART B (50 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B26. Find the absolute minimum and maximum values of $f(x) = \ln x - \dfrac{x^2}{2}$ on the interval $[e^{-1}, e^2]$. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

7 marks B27. A wire 4 meters long is cut into two pieces. One piece is bent into a square for a frame for a stained glass ornament, while the other piece is bent into a circle for a TV antenna. To reduce storage space, where should the wire be cut to minimize the total area, which is the area of the square plus the area of the circle? Give the length of wire used for the square and the circle. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

7 marks B28. Find the volume of the solid obtained by rotating the region bounded by the curves $xy = 2$ and $y = 3 - x$ about the x-axis. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B29. Gravel is being dumped from a conveyor belt at a rate of 12 m^3/min. It forms a pile in the shape of a right circular cone whose base diameter and height are always equal to each other. How fast is the height of the pile increasing when the pile is 12 meters high? [Hint: The volume of a right circular cone with base radius r and height h is $V = \frac{\pi}{3}r^2h$.]

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B30. Sketch the region enclosed by the graphs of the functions $f(x) = x^3$ and $g(x) = 3x$. Find the area of this region. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

B31. Evaluate the following indefinite integrals:

4 marks (a) $\displaystyle\int \frac{e^x}{1+e^x}\,dx$

4 marks (b) $\displaystyle\int x\sqrt{1-x^2}\,dx.$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

B32. Find the following limits:

5 marks (a) $\lim_{x\to 0} (1+\sin(\pi x))^{\frac{1}{x}}$

5 marks (b) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

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