

1. Which (if any) of the following functions is an odd function?

(A) $f(x) = e^{x^3-x}$ (B) $f(x) = x^2 \sin x$ (C) $f(x) = \ln x$

(D) $f(x) = \sqrt{x-2}$ (E) none of (A) to (D)

2. $\frac{d}{dx} \log_2(x 2^x) =$

(A) $1 + \frac{1}{x \ln 2}$ (B) $\frac{1}{x \ln 2}$ (C) 1 (D) $x + \frac{1}{x}$ (E) none of (A) to (D)

3. If $f(x) = \tan x$, then the following is a critical number (also called critical point) of $f(x)$:

(A) 0 (B) $\pi/2$ (C) $-\pi/2$ (D) $\pi/4$ (E) none of (A) to (D)

4. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x} =$

(A) $-\infty$ (B) ∞ (C) 0 (D) 1 (E) does not exist

5. If $\lim_{x \rightarrow -3} \frac{f(x)}{x+3} = 7$, then $\lim_{x \rightarrow -3} f(x)$ is equal to

(A) $7/3$ (B) $3/7$ (C) 0 (D) -3 (E) ∞

6. $\lim_{x \rightarrow 3} \frac{\int_3^x e^{s^3} ds}{x-3} =$

(A) 1 (B) e^9 (C) e (D) $e^4/2$ (E) e^{27}

7. $\lim_{x \rightarrow -\infty} x e^x =$

(A) -1 (B) 0 (C) ∞ (D) 1 (E) none of (A) to (D)

8. The function $f(x) = \sqrt{|x|}$ defined on the real axis

- (A) is differentiable everywhere
- (B) has a vertical asymptote
- (C) has an absolute minimum
- (D) has an absolute maximum
- (E) none of (A) to (D)

9. $\int_1^3 \frac{\ln x}{x} dx =$

- (A) $\ln 3$ (B) 0 (C) $\frac{(\ln 3)^2}{2}$ (D) $\ln 3 - 1$ (E) none of (A) to (D)

10. If $f(x)$ is a continuous function on the interval $[a, b]$, where $a < b$, then the function $F(x) = \int_a^x ((f(s))^2 + 1) ds$

- (A) is continuous but not differentiable on $[a, b]$;
- (B) is differentiable but not continuous on $[a, b]$;
- (C) is continuous, differentiable and decreasing on $[a, b]$;
- (D) is continuous, differentiable and increasing on $[a, b]$;
- (E) none of (A) to (D)

11. The range of the function $f(x) = e^{-e^{-x}}$ is

- (A) $(0, 1)$ (B) $(0, \infty)$ (C) $(-1, 0)$ (D) $(-\infty, \infty)$ (E) none of (A) to (D)

12. $\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \frac{2i}{n}$ equals

- (A) $\int_{-1}^1 t dt$ (B) $\int_0^2 t dt$ (C) $\int_0^2 2t dt$ (D) $\int_0^1 dt$ (E) none of (A) to (D)

13. $\frac{d}{dx} \int_1^{\sqrt{x}} e^{t^2} dt =$

- (A) e^x (B) $e^x - 1$ (C) $\frac{e^{x^2}}{\sqrt{x}}$ (D) $\frac{e^x}{2\sqrt{x}}$ (E) none of (A) to (D)

14. $\int_0^{\sqrt{3}} \frac{dt}{1+t^2} =$

- (A) $\pi/3$ (B) $\pi/6$ (C) $\pi/2$ (D) π (E) none of (A) to (D)

15. $\int_0^1 \sqrt[3]{1+7x} dx =$

- (A) $28/15$ (B) 1 (C) $45/28$ (D) $9/28$ (E) none of (A) to (D)

16. $\int \sec^2 x \tan^3 x dx =$

- (A) $\frac{\tan^4 x}{4} + C$ (B) $\tan^4 x - 1 + C$ (C) $\frac{\tan^2 x}{2} + C$

- (D) $-\frac{\tan^3 x}{3} + C$ (E) none of (A) to (D)

17. $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} =$

- (A) 1 (B) 2 (C) e^2 (D) e (E) none of (A) to (D)

18. $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}} =$

- (A) 0 (B) 1 (C) e^3 (D) e (E) none of (A) to (D)

19. If $f''(x) = 2 + \cos x$, $f(0) = -1$, $f(\pi/2) = 0$, then $f(x) =$

- (A) $-\cos x$ (B) $x^2 - \pi x/2 - \cos x$ (C) $2x^2 - \pi x - \cos x$ (D) $\cos x + x^2$ (E) none of (A) to (D)

20. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{2^i} =$

- (A) 1 (B) 2 (C) 1/2 (D) 4 (E) does not exist

21. If $y = \ln(x^2 + y^2)$, then $\frac{dy}{dx}$ at the point $(1, 0) =$

- (A) 1 (B) 0 (C) 2 (D) $e/2$ (E) none of (A) to (D)

22. If $f(x)$ has a continuous derivative on $[1, 7]$, $f'(1) = 3$ and $f'(7) = -5$, which of the following statements is necessarily true?

- (A) $f(x) = 0$ has at most one solution between 1 and 7
(B) $f(x) = 0$ has at least one solution between 1 and 7
(C) the graph of $f(x)$ has a vertical asymptote between 1 and 7
(D) the graph of $f(x)$ has a horizontal tangent between 1 and 7
(E) none of (A) to (D)

23. If $f(2) = -6$, f' is continuous and $\int_2^9 f'(x) dx = 17$, then $f(9) =$

- (A) 11 (B) 23 (C) 77 (D) 45 (E) none of (A) to (D)

24. $\int_0^1 \frac{e^t + 1}{e^t + t} dt =$

- (A) 1 (B) $\ln(e + 1)$ (C) e (D) $1 - e$ (E) none of (A) to (D)

25. If $y(x) = x^x$, then $y'(x) =$

- (A) xx^{x-1} (B) $x^x \ln x$ (C) x^{x-1} (D) $x^x(\ln x + 1)$ (E) none of (A) to (D)

26. If the derivative of a function f is $f'(x) = (x - 2)^2(x + 4)^7x^4$ then f is decreasing on the interval

- (A) $(0, 2)$ (B) $(2, \infty)$ (C) $(-\infty, -4)$ (D) $(-4, 0)$ (E) none of (A) to (D)

27. $\int_0^4 |\sqrt{x} - 1| dx =$

- (A) -2 (B) 2 (C) 4 (D) -4 (E) none of (A) to (D)

28. $\int_0^4 \sqrt{16 - x^2} dx =$

- (A) 2π (B) 8π (C) 4π (D) 16π (E) none of (A) to (D)

29. The tangent line to the curve $y = \frac{e^x}{x}$ at the point $(1, e)$ has the equation

- (A) $y = e$ (B) $y = e + ex$ (C) $y = e - ex$ (D) $y = 0$ (E) none of (A) to (D)

30. The area of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ is

- (A) $1/2$ (B) $1/6$ (C) 1 (D) $1/3$ (E) none of (A) to (D)

31. $\int \sin x \cos(\cos x) dx =$

- (A) $\cos(\sin x) + C$ (B) $-\sin(\cos x) + C$ (C) $\sin(\cos x) + C$
(D) $-\cos(\sin x) + C$ (E) none of (A) to (D)

32. $\int_{-\pi}^{\pi} (x^2 + 1) \sin x dx =$

- (A) $\pi^2 + 1$ (B) $4\pi^2 + 1/2$ (C) 0 (D) 2π (E) none of (A) to (D)

33. Find the absolute maximum and minimum values of $f(x) = xe^{-x^2/8}$ on the interval $[-1, 4]$.
34. Evaluate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \left(\frac{k}{n}\right)^9$ by evaluating an integral.
35. Calculate the integral $\int_0^a x\sqrt{a^2 - x^2} dx$.
36. A hare and a tortoise start moving from the same point. The hare travels north at 1.5 km/hour and the tortoise travels east at 2 km/hour. At what rate is the distance between the hare and the tortoise increasing 2 hours later?
37. Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant. Note: the first quadrant is the region in the xy plane where x and y are both positive.
38. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $x = y^2$ about the line $y = 1$.